

# Math 124, Fall 2021 Midterm II

November 16, 2021

Name \_\_\_\_\_

Student Number \_\_\_\_\_

## Instructions.

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ( $\frac{2\ln 3}{\pi}$  and  $1/3$  are exact, 0.699 and 0.333 are approximations for the those numbers.)
- **Show your work.** If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**
- If you continue a question on the last page, make a note for me. If you have read all the instructions, put a smiley next to your student number for a bonus point.

1. (13 points) Differentiate the following functions. You do not have to simplify your answers, but make sure your parentheses are correct. In part (c), you need to show your work.

(a)  $f(x) = \ln(5x^2 + 1) - 12e^{\cos x} + \frac{\arctan(x^3)}{5}$

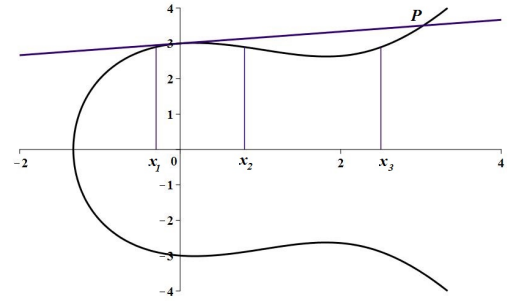
(b)  $g(t) = \sqrt{3t + \sqrt{5t + 8}}$

(c)  $h(x) = (1 + \sin^2 x)^{\cos x}$

2. (15 points) The line is tangent to the curve

$$y^2 = x^3 - 3x^2 + x + 9$$

at the point  $(0, 3)$  as shown on the right.



- (a) Find the  $x$ -coordinate of the second point  $P$  where the tangent line intersects the curve. Use the graph as a guide and to check your answer approximately.

- (b) If the point  $(a, 2.9)$  is on the curve, use linear approximation to approximate a value of  $a$ .

- (c) As show on the picture, there are three values of  $x$  that correspond to  $y = 2.9$ . Which of the three did you approximate above? Was your approximation an overestimate or an underestimate?

- (d) Find the value of  $y''$  for the curve at the point  $(0, 3)$ .

3. (13 points) A particle is traveling on the  $xy$ -plane with parametric equations

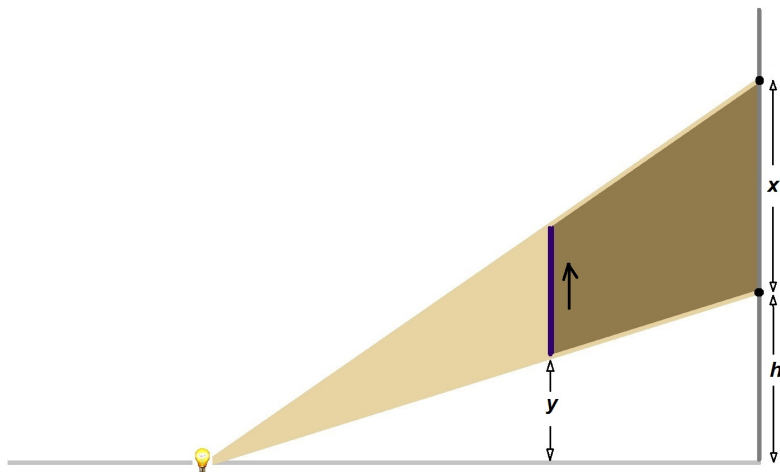
$$x = t^2 + 5t + 2 \quad \text{and} \quad y = t^3 - 3t + 1.$$

(a) At what points  $(x, y)$  is the tangent line horizontal?

(b) Compute  $\frac{d^2y}{dx^2}$  as a function of  $t$ . Simplify your answer.

(c) At  $t = 1$  the particle leaves its path and starts going with constant speed with linear equations  $x = a + bt$  and  $y = c + dt$ . At the moment it leaves its path, *its horizontal and vertical velocities remain the same*. Find the new linear equations for the position of the particle for  $t \geq 1$ .

4. (9 points) A light source is located 20 meters from a wall. A 4 meter high billboard hangs 12 meters from the wall and it is rising up at a rate of 0.3 meters per second. Below is a side view. The billboard casts a shadow on the wall as shown. Let  $y$  represent the height of the lowest point of the billboard from the ground as shown.
- Note: The figure is not to scale.



- (a) Let  $h$  be the position of the bottom of the shadow measured from the ground as shown. How fast is  $h$  changing when the lowest part of the billboard is 2 meters from the ground?
- (b) Let  $x$  be the length of the shadow as shown. How fast is  $x$  changing when the lowest part of the billboard is 2 meters from the ground?