Math 124, Fall 2021 Solutions to Midterm I

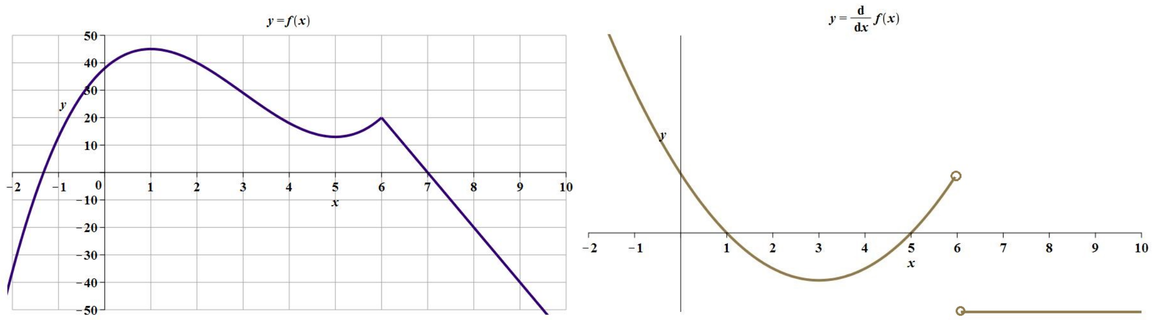
1. (a)

$$\frac{d}{dx}\left(5x^3e^x - 4x^{-3} + 2\sqrt{2}x^{1/2}\right) = 15x^2e^x + 5x^3e^x + 12x^{-4} + \sqrt{2}x^{-1/2}$$

(b)

$$g'(t) = \frac{(\cos t - t\sin t - \cos t)(5t^2+4t+1) - (t\cos t - \sin t)(10t+4)}{(5t^2+4t+1)^2}$$

(c)



$$f'(8) < f'(3) < f'(1) < f'(0) < f'(-1)$$

2. (a) $\displaystyle\lim_{x\to 3^+}\frac{x^2-7x+12}{x^2-6x+9} = \lim_{x\to 3^+}\frac{(x-3)(x-4)}{(x-3)(x-3)} = \lim_{x\to 3^+}\frac{x-4}{x-3} = -\infty$

(b) $\displaystyle\lim_{x\to 0}\frac{\sin^2(5x)}{7x} = \lim_{x\to 0}\frac{5}{7}\cdot\frac{\sin(5x)}{5x}\cdot\sin(5x) = \frac{5}{7}\cdot\lim_{x\to 0}\frac{\sin(5x)}{5x}\cdot\lim_{x\to 0}\sin(5x) = \frac{5}{7}\cdot 1\cdot 0 = 0$

(c)

$$\lim_{h\to 0}\frac{\sqrt{x+h+3}-\sqrt{x+3}}{h}\cdot\frac{\sqrt{x+h+3}+\sqrt{x+3}}{\sqrt{x+h+3}+\sqrt{x+3}} = \lim_{h\to 0}\frac{x+h+3-(x+3)}{h\left(\sqrt{x+h+3}+\sqrt{x+3}\right)}$$

$$= \lim_{h\to 0}\frac{1}{\left(\sqrt{x+h+3}+\sqrt{x+3}\right)} = \frac{1}{2\sqrt{x+3}}$$

3. (13 points) Let

$$f(x) = \frac{x^2 - 5x + 10}{x + 1}.$$

(a)

$$f'(x) = \frac{(2x-5)(x+1) - (x^2 - 5x + 10) \cdot 1}{(x+1)^2} = \frac{x^2 + 2x - 15}{(x+1)^2} = 0$$

when

$$x^2 + 2x - 15 = (x+5)(x-3) = 0$$

so at $x = -5$ and $x = 3$.

(b)

$$f'(4) = \frac{16 + 8 - 15}{5^2} > 0$$

so the function is increasing at $x = 4$.

(c) $f'(1) = -2$ and $f(1) = 3$ so the tangent line is

$$y - 3 = -3(x - 1)$$

(d)

$$\lim_{x \to -1^+} \frac{x^2 - 5x + 10}{x + 1} = \infty$$

$$\lim_{x \to -1^-} \frac{x^2 - 5x + 10}{x + 1} = -\infty$$

so the graph has a vertical asymptote at $x = -1$.

$$\lim_{x \to \infty} \frac{x^2 - 5x + 10}{x + 1} = \infty$$

$$\lim_{x \to -\infty} \frac{x^2 - 5x + 10}{x + 1} = -\infty$$

so the graph has no horizontal asymptotes.

4. The tangent to the graph of

$$y = \frac{5}{2x - 3}, \qquad y(4) = \frac{5}{5} = 1$$

$$y' = -\frac{10}{(2x-3)^2}, \qquad y'(4) = -\frac{10}{25} = -\frac{2}{5}$$

The tangent line has equation

$$y - 1 = -\frac{2}{5}(x - 4)$$

To get the y intercept, set $x = 0$:

$$y - 1 = -\frac{2}{5}(-4)$$

so $y = \frac{13}{5}$.

To get the x intercept, set $y = 0$:

$$0 - 1 = -\frac{2}{5}(x - 4)$$

so $x = \frac{13}{2}$.

The area of the triangle is

$$A = \frac{1}{2} \cdot \frac{13}{2} \cdot \frac{13}{5} = \frac{169}{20} = 8.45$$