

Math 124, Fall 2021 Midterm I

October 26, 2021

Name _____

Student Number _____

Instructions.

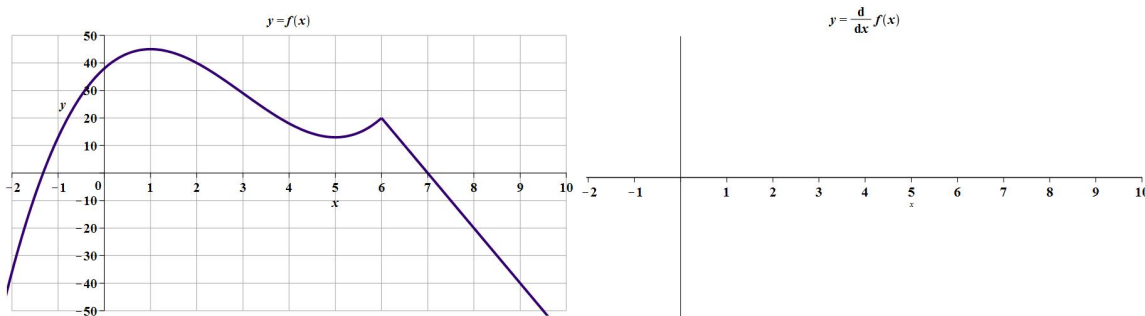
- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for the those numbers.)
- **Show your work.** If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.** If you continue a question on the last page, make a note for me.

1. (14 points) Answer the following.

- (a) Differentiate $h(x) = 5x^3 e^x - \frac{4}{x^3} + \sqrt{8x}$.
 You do not need to simplify your answer.

- (b) Differentiate $g(t) = \frac{t \cos t - \sin t}{5t^2 + 4t + 1}$.
 You do not need to simplify your answer, but be careful with your use of parentheses.

(c) A graph of $y = f(x)$ is given.



(i) Use the graph arrange the following in *increasing order* from smallest to largest:

$$f'(-1), f'(0), f'(1), f'(3), f'(8)$$

(ii) On the axis provided on the right, sketch a graph of the derivative of $y = f(x)$. Note that the y -axis is not scaled. I am interested in the *shape* of the derivative graph and not on the actual values of y' . If there is a place where the derivative does not exist, use a \circ on your graph.

2. (12 points) Evaluate the following limits. Show your reasoning through algebra. You can use your calculator to check your answers, but you cannot use numerical values to justify your answers.

(a) $\lim_{x \rightarrow 3^+} \frac{x^2 - 7x + 12}{x^2 - 6x + 9}$

(b) $\lim_{x \rightarrow 0} \frac{\sin^2(5x)}{7x}$

- (c) Use the *limit definition of the derivative* to compute the derivative $f'(x)$ for $f(x) = \sqrt{x+3}$.

3. (13 points) Let

$$f(x) = \frac{x^2 - 5x + 10}{x + 1}.$$

(a) Find all values of x where the graph of $y = f(x)$ has a horizontal tangent line.

(b) Is the function increasing or decreasing at $x = 4$? Explain.

(c) Find the equation of the tangent line to the graph of $y = f(x)$ at the point where $x = 1$.

(d) Does the graph of $y = f(x)$ have any horizontal or vertical asymptotes? Explain.

4. (11 points) The tangent to the graph of

$$y = \frac{5}{2x - 3}$$

at the point where $x = 4$ cuts a triangle in the first quadrant as shown. Find the area of the triangle.

