Solutions to Math 124 F Winter 2023 Midterm II

1. (a) $f'(x) = 4(e^x + 5x^4)\cos(e^x + x^5) + \dfrac{\sec^2 x}{2\sqrt{1+\tan x}} - \dfrac{2}{x(3+2\ln x)^2}$

   (b) $g(x) = \left(9+8x^7\right)^{(6x^5+4)}$

$$\ln(g(x)) = \ln\left(\left(9+8x^7\right)^{(6x^5+4)}\right) = (6x^5+4)\ln\left(9+8x^7\right)$$

$$\frac{g'(x)}{g(x)} = (30x^4)\ln(9+8x^7) + \frac{56x^6(6x^5+4)}{(9+8x^7)}$$

$$g'(x) = \left((30x^4)\ln(9+8x^7) + \frac{56x^6(6x^5+4)}{(9+8x^7)}\right)\cdot(9+8x^7)^{(6x^5+4)}$$

   (c) If you simplify before you differentiate:

$$h(x) = \ln\left(\sqrt{\frac{9+x^2}{9-x^2}}\right) = \frac{1}{2}\ln(9+x^2) - \frac{1}{2}\ln(9-x^2)$$

$$h'(x) = \frac{2x}{2(9+x^2)} - \frac{2x}{2(9-x^2)} = \frac{18x}{81-x^4}$$

   If you differentiate using the Chain Rule and the Quotient Rule:

$$h'(x) = \frac{1}{\sqrt{\frac{9+x^2}{9-x^2}}}\cdot\frac{1}{2\sqrt{\frac{9+x^2}{9-x^2}}}\cdot\frac{2x(9-x^2)+2x(9+x^2)}{(9-x^2)^2} = \frac{1}{2\left(\frac{9+x^2}{9-x^2}\right)}\cdot\frac{36x}{(9-x^2)^2} = \frac{(9-x^2)}{2(9+x^2)}\cdot\frac{36x}{(9-x^2)^2} = \frac{18x}{81-x^4}$$

2. (a) From $v = \omega r$ we get $\omega = \frac{12}{24} = 0.5$. So,

$$x(t) = 24\cos(0.5t) \qquad y(t) = 24\sin(0.5t).$$

   (b) For the slope we need:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{12\cos(0.5t)}{-12\sin(0.5t)}$$

   When $t = 4$,

$$\frac{dy}{dx} = \frac{12\cos(2)}{-12\sin(2)} = -\frac{\cos(2)}{\sin(2)}$$

   For the point we need:

$$x(4) = 24\cos(2) \qquad y(4) = 24\sin(2).$$

   So, the tangent line is:

$$y - 24\sin(2) = -\frac{\cos(2)}{\sin(2)}(x - 24\cos(2))$$

   (c) The giraffe is on the x-axis where $y = 0$ so we solve:

$$0 - 24\sin(2) = -\frac{\cos(2)}{\sin(2)}(x - 24\cos(2))$$

   Which gives $x = \frac{24}{\cos 2}$. Note that this is a negative number. The distance then is,

$$24 - \frac{24}{\cos 2} \approx 81.672$$

3. (a) Partial derivative:

$$3x^2 - 8xy - 4x^2y' + 32y^3y' = 0$$

At $(2, 1)$ the slope is $y' = 1/4$. Therefore, the tangent line is $y - 1 = \frac{1}{4}(x - 2)$.

(b) Differentiate again to get:

$$6x - 8(y + xy') - 4(2xy' + x^2y'') + 32(3y^2y'y' + y^3y'') = 0$$

When $x = 2$, $y = 1$, and $y' = 1/4$

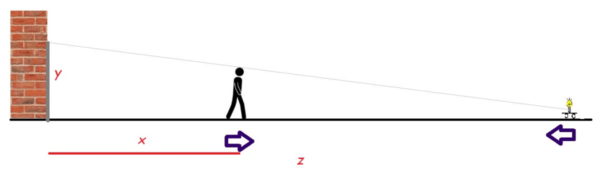
$$12 - 8(1 + \frac{1}{2}) - 4(1 + 4y'') + 32(\frac{3}{16} + y'') = 0$$

which gives $y'' = -1/8 < 0$.

(c) The function is increasing concave down so the possible picture is A.

4. The rates in the questions are

$$\frac{dx}{dt} = 1.3, \frac{dz}{dt} = -0.7, \frac{dy}{dt} = ?$$



Similar triangles to relate the quantities

$$\frac{y}{1.8} = \frac{z}{z - x}$$

or $1.8z = y(z - x)$.

Differentiate

$$1.8\frac{dz}{dt} = \frac{dy}{dt}(z - x) + y\left(\frac{dz}{dt} - \frac{dx}{dt}\right)$$

When $x = 12$ and $z = 24$ we get $y = 3.6$, so

$$1.8(-0.7) = \frac{dy}{dt}(12) + 3.6\,(-0.7 - 1.3)$$

so $\frac{dy}{dt} = 0.495$ meters per second.