

# Math 124 F Winter 2023 Midterm II

February 21, 2023

Name \_\_\_\_\_

Student Number \_\_\_\_\_

## Instructions

- These exams will be scanned. **Please write your name and student number clearly.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting. **Hand in your notes with your exam.**
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ( $\frac{2\ln 3}{\pi}$  and  $1/3$  are exact, 0.699 and 0.333 are approximations for those numbers.)
- Show your work. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. (16 points) Differentiate the following functions. Make sure your use of parentheses is correct.

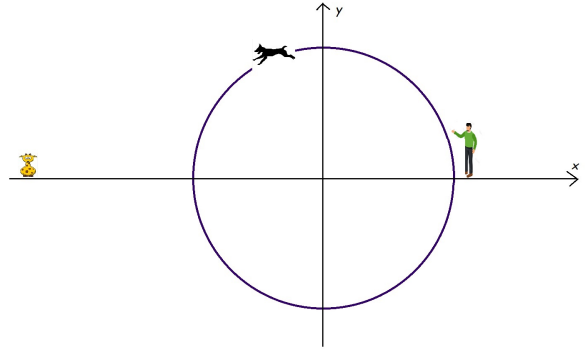
(a)  $f(x) = 4 \sin(e^x + x^5) + \sqrt{1 + \tan x} + \frac{1}{3 + 2 \ln x}$

(b)  $g(x) = (9 + 8x^7)^{(6x^5+4)}$

(c)  $h(x) = \ln \left( \sqrt{\frac{9 + x^2}{9 - x^2}} \right)$

*Simplify your answer here so you have no negative exponents and at most one fraction line in your answer.*

2. (13 points) Blaise takes his dog Pascal off his leash at  $t = 0$  and Pascal starts running counter-clockwise around a circular path of radius 24 meters at a speed of 12 meters per second. At  $t = 4$  seconds, Pascal spots his favorite toy giraffe and starts on a path tangent to the circle to get to it.



*Note: The figure is not to scale.*

- (a) Write down parametric equations for Pascal's motion on the circular path for  $0 \leq t \leq 4$ , where a coordinate system is imposed with the center of the circular path at the origin as shown.

- (b) Write down the equation of the tangent line, Pascal's path to his toy giraffe.

- (c) What is the distance between the giraffe and Blaise?  
*Approximate your final answer here to two digits after the decimal.*

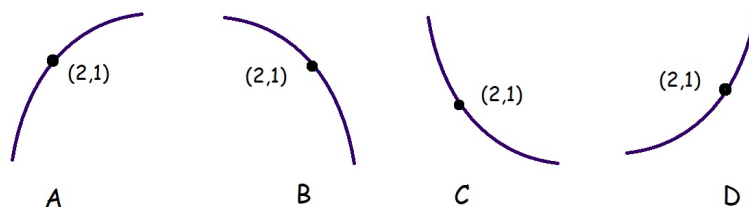
3. (14 points) A curve is given implicitly by the equation

$$x^3 - 4x^2y + 8y^4 = 0$$

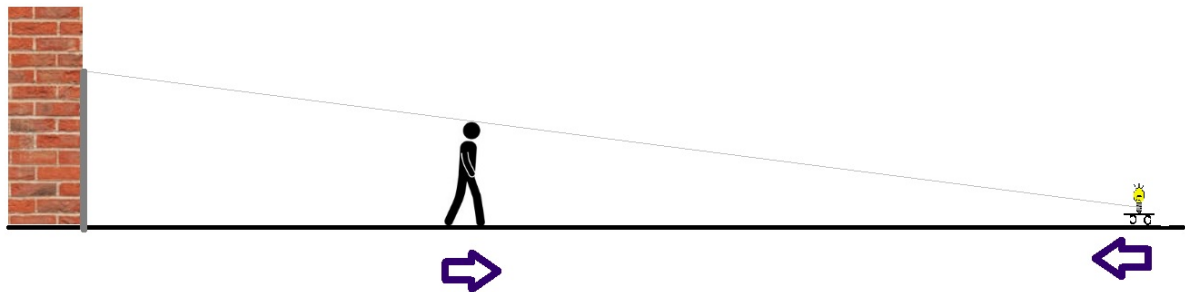
(a) Find the equation of the tangent line to this curve at the point  $(2, 1)$

(b) Find the value of  $y''$  at the point  $(2, 1)$ .

(c) Use your answers to parts (a) and (b) to decide which one of these could be the picture of the curve near the point  $(2, 1)$ .



4. (7 points) Omar, who is 180 centimeters tall, is walking away from a wall at a speed of 1.3 meters per second. A light source on wheels is moving towards the wall at a speed of 0.7 meters per second. How fast is the height of Omar's shadow on the wall increasing when he is 12 meters from the wall and exactly halfway between the wall and the lamp?
- Note: The figure is not to scale. You can ignore the height of the lamp and assume it is on the ground.*



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