Math 124 F, Winter 2023 Midterm I

January 23, 2023

Name_____

Student Number_____

Instructions

- These exams will be scanned. **Please write your name and student number clearly.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for those numbers.)
- Show your work. If we cannot read or follow your work, we cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. The parts of this question are not related.

   (a) (5 points) Evaluate $\lim_{x \to 2} \dfrac{x^3 - 4x}{x^2 + x - 6}$.

   (b) (5 points) Evaluate $\lim_{t \to \frac{\pi}{2}} \dfrac{\sqrt{\sin^2 t + a \cos^2 t} - \sin t}{\cos^2 t}$. (Your answer will depend on a.)

   (c) (2 points) Can a function have three horizontal asymptotes? Why or why not?

2. The parts of this question are not related.

(a) (6 points) Compute $f'(x)$ for $f(x) = \dfrac{2e^x}{3} + \sqrt{\dfrac{4}{x}} - \dfrac{5}{6x^2} + 7x^e$.

(b) (7 points) Find the equation of the tangent line to $g(x) = \dfrac{2x^3 + 4\tan x}{5x^6 + 7\cos x}$ at the point where $x = 0$.

(c) (3 points) Compute $\lim\limits_{x \to a} \dfrac{\tan x - \tan a}{x - a}$. (Your answer will depend on a.)

3. (11 points) Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} \dfrac{x^3 - 27}{x - 3}, & x < 3 \\ \dfrac{bx + 1}{2x - 1} + a, & x \geq 3. \end{cases}$$

is differentiable (and hence continuous) at $x = 3$.

4. (11 points) Find equations of the two tangent lines to $y = x^3 - x + 1$ that pass through the point $(-2, 3)$. Note that $(-2, 3)$ is not on the curve.

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