Solutions to Math 124 C Spring 2023 Midterm I

1. Differentiate the following functions. You do not have to simplify your answers, but make sure your use of parentheses is correct.

   (a) $f(x) = 0.3x^2 - \dfrac{1}{2x^3} + e^{2x} + 3\sqrt{x} + \pi$

   $f'(x) = 0.6x + \dfrac{3}{2x^4} + 2e^{2x} + \dfrac{3}{2\sqrt{x}}$

   (b) $g(x) = \dfrac{5\sin(6x) + 7\cos(8x)}{9e^x - x^{10}}$

   $g'(x) = \dfrac{(30\cos(6x) - 56\sin(8x))(9e^x - x^{10}) - (5\sin(6x) + 7\cos(8x))(9e^x - 10x^9)}{(9e^x - x^{10})^2}$

   (c) $h(x) = (3x^3 + 5)\tan(e^x)$

   $h'(x) = (9x^2)\tan(e^x) + e^x(3x^3 + 5)\sec^2(e^x)$

2. (a) (i) $\displaystyle\lim_{h\to 0} \frac{\sqrt{(5+h)^2 - 16} - 3}{h} \cdot \frac{\sqrt{(5+h)^2 - 16} + 3}{\sqrt{(5+h)^2 - 16} + 3} = \lim_{h\to 0} \frac{(5+h)^2 - 25}{h\left(\sqrt{(5+h)^2 - 16} + 3\right)}$

   $\displaystyle = \lim_{h\to 0} \frac{h(10+h)}{h\left(\sqrt{(5+h)^2 - 16} + 3\right)} = \lim_{h\to 0} \frac{10+h}{\sqrt{(5+h)^2 - 16} + 3} = \frac{10}{\sqrt{5^2 - 16} + 3} = \frac{10}{3+3} = \frac{5}{3}$

   (ii) $f(x) = \sqrt{x^2 - 16}$, $a = 5$, $f'(x) = \dfrac{x}{\sqrt{x^2 - 16}}$, $f'(a) = f'(5) = \dfrac{5}{\sqrt{25 - 16}} = \dfrac{5}{3}$

   (b) $\displaystyle\lim_{x\to 5} \frac{x^2 - 25}{x^2 - 9x + 20} = \lim_{x\to 5} \frac{(x-5)(x+5)}{(x-5)(x-4)} = \lim_{x\to 5} \frac{x+5}{x-4} = 10$

   OR

   $\displaystyle\lim_{x\to 5} \frac{x^2 - 25}{x^2 - 9x + 20} \stackrel{\text{LH}}{=} \lim_{x\to 5} \frac{2x}{2x - 9} = \frac{10}{1} = 10$

3. (a) The point of tangency is $(0, y(0)) = (0, 1)$ and from

   $$y' = e^x(x^2 + x + 1) + e^x(2x + 1) = e^x(x^2 + 3x + 2).$$

   the slope is $y'(0) = 2$ so the tangent line L_1 is given by

   $$y - 1 = 2(x - 0)$$

   or $y = 2x + 1$.

   (b) The tangent line L_2 must have slope $-\frac{1}{2}$ since it is perpendicular to L_1, so

   $$y' = -2x + \frac{3}{2} = -\frac{1}{2}$$

   so the point of tangency has $x = 1$ and $y = -1 + \frac{3}{2} + 7 = \frac{15}{2}$. Therefore, the equation of the tangent line L_2 is

   $$y - \frac{15}{2} = -\frac{1}{2}(x - 1)$$

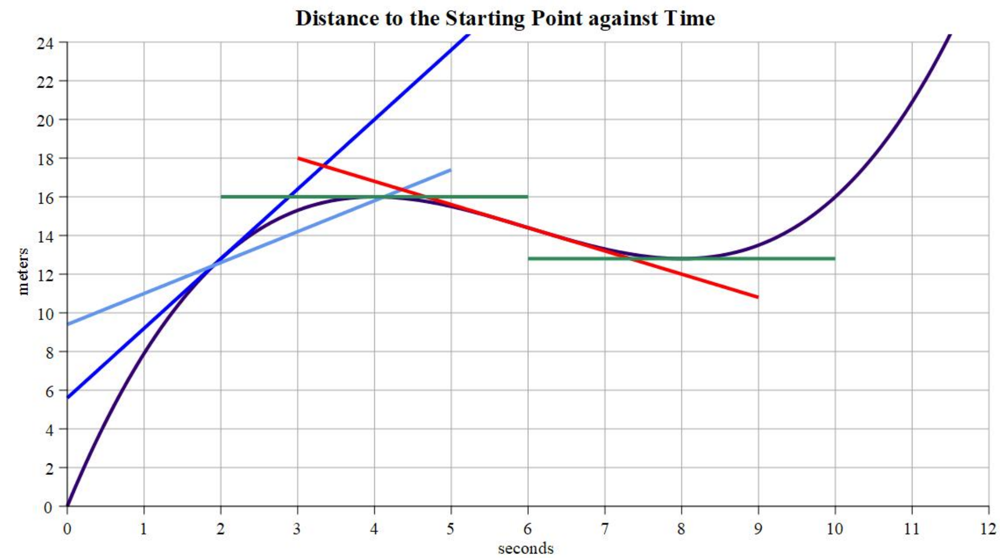
   or $y = -\frac{1}{2}x + 8$.

   (c) Solve

   $$2x + 1 = -\frac{1}{2}x + 8$$

   to get $x = \frac{14}{5} = 2.8$ and $y = 2 \cdot \frac{14}{5} + 1 = \frac{33}{5} = 6.6$

4. This was not graded for work shown, but the parts needed some lines to be drawn for the answers.



(a) The velocity of the object at $t = 2$ (See blue tangents above)

(b) -1.2 (See red tangent above)

(c) $t = 4, 8$ (See green tangents above)

(d) $(4, 8)$

(e) $s'(6) < s'(4) < s'(9.4) < s'(0.5)$ (See orange tangents below)