Math 124 C Fall 2022 Midterm II

November 15, 2022

Name_____

Student Number_____

Instructions

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**

- There are 4 questions. The exam is out of 50 points.

- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.

- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and 1/3 are exact, 0.699 and 0.333 are approximations for those numbers.)

- **Show your work**. If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. Compute $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for the following.

   (a) (6 points) $y = \sin\left(x^3 + e^{4x}\right) + \ln(x^2 + 1)$

   (b) (6 points) $x = 3t^2 + e^t$ and $y = 5t + \cos t$

2. Find the equation of the tangent line to $y = f(x)$ at $x = 3$ for the following.

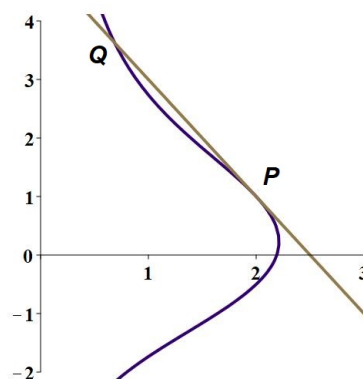(a) (7 points) $y = \sqrt{2x + \sqrt{x + 6}}$

(b) (6 points) $y = (x - 2)^{x^2 - 8}$

3. (15 points) The line is tangent to curve given implicitly by

$$4xy^2 - 4y + 5x^2 = 24$$

at the point $P(2, 1)$. It intersects the curve again at the point Q as shown.

(a) Find the equation of the tangent line shown.

(b) Find the x-coordinate of the point Q.

(c) Use *linear approximation* to approximate the y-coordinate of the point Q.

4

4. (10 points) A tank is in the shape of an inverted cone of radius 3 meters and height 7 meters. It is being filled at a rate of 2 cubic meters per minute. How fast is the water level rising when the tank is filled to half its capacity?
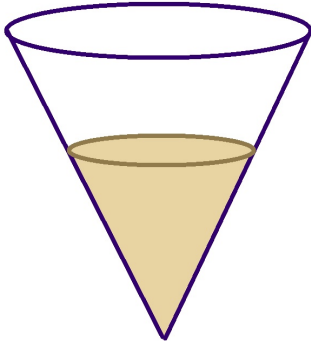
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