Math 124 C Fall 2022 Solutions to Midterm I

1. (a)

$$\lim_{x\to\infty}\frac{\sqrt{9+x}-3}{4-\sqrt{16+2x}}=\lim_{x\to\infty}\frac{\frac{\sqrt{9+x}-3}{\sqrt{x}}}{\frac{4-\sqrt{16+2x}}{\sqrt{x}}}=\lim_{x\to\infty}\frac{\sqrt{\frac{9+x}{x}}-\frac{3}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-\sqrt{\frac{16+2x}{x}}}=\lim_{x\to\infty}\frac{\sqrt{\frac{9}{x}+1}-\frac{3}{\sqrt{x}}}{\frac{4}{\sqrt{x}}-\sqrt{\frac{16}{x}+2}}=\frac{\sqrt{1}-0}{0-\sqrt{0+2}}=-\frac{1}{\sqrt{2}}$$

(b)

$$\lim_{h\to0}\frac{f(3+h)-f(3)}{h}=\lim_{h\to0}\frac{\sqrt{2(3+h)+1}-\sqrt{7}}{h}=\lim_{h\to0}\frac{\sqrt{2h+7}-\sqrt{7}}{h}\cdot\frac{\sqrt{2h+7}+\sqrt{7}}{\sqrt{2h+7}+\sqrt{7}}$$

$$=\lim_{h\to0}\frac{2h+7-7}{h\left(\sqrt{2h+7}+\sqrt{7}\right)}=\lim_{h\to0}\frac{2h}{h\left(\sqrt{2h+7}+\sqrt{7}\right)}=\lim_{h\to0}\frac{2}{\left(\sqrt{2h+7}+\sqrt{7}\right)}=\frac{2}{\left(\sqrt{7}+\sqrt{7}\right)}=\frac{1}{\sqrt{7}}$$

(c)

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin\left(\frac{\pi}{6}\right)-\sin x}{\frac{\pi}{6}-x}=\frac{d}{dx}\sin x\Big|_{x=\frac{\pi}{6}}=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$$

2. (a) $f'(x)=\dfrac{3e^x}{5}+\dfrac{1}{\sqrt{x}}+\dfrac{6}{x^3}+ex^{e-1}+13.$

(b) $g'(x)=\dfrac{48x^5\left(5+4\sqrt[3]{x}\right)-\left(8x^6+7\right)\left(\frac{4}{3}x^{-2/3}\right)}{\left(5+4\sqrt[3]{x}\right)^2}$ $\quad g'(1)=\dfrac{48(5+4)-(8+7)\left(\frac{4}{3}\right)}{(5+4)^2}=\dfrac{412}{81}$

(c) The parabola and the tangent line share the point $(1,1)$ and the slope 5:

$$1=A\cdot1^2+B$$

and

$$y'(1)=2A\cdot1=5$$

So $A=5/2$ and $B=1-5/2=-3/2$.

3. Since $f(x)=\dfrac{x}{x^2+x-6}=\dfrac{x}{(x+3)(x-2)}$

(a)

$$\lim_{x\to2^+}f(x)=\infty\qquad\lim_{x\to2^-}f(x)=-\infty\qquad\lim_{x\to\infty}f(x)=0\qquad\lim_{x\to-\infty}f(x)=0$$

and

$$f'(x)=\frac{x^2+x-6-x(2x+1)}{(x^2+x-6)^2}\qquad f'(0)=-\frac{1}{6}$$

(b) The graph of the function $y=f(x)$ is given by picture **D**, matching the vertical asymptote $x=2$, the horizontal asymptote $y=0$ and the negative derivative at $x=0$.

4. We write the slope of the tangent at the point $(a,a^2-6a+11)$in two ways:

$$2a-6=\frac{a^2-6a+11-7}{a-0}$$

giving us the quadratice equation

$$a^2-4=0.$$

Since $a>0$ in the picture, we have $a=2$. So the slope is $2a-6=-2$, the y-coordinate is $a^2-6a+11=3$, the tangent line is

$$y-3=-2(x-2),$$

which has x-intercept $x=7/2$, giving the area of the triangle to be $49/4=12.25$.