Math 124 C Fall 2022 Midterm I

October 25, 2022

Name_____

Student Number_____

Instructions

- **These exams will be scanned. Please write your name and student number clearly for easy recognition.**
- There are 4 questions. The exam is out of 50 points.
- You are allowed to use one page of notes written only on one side of the sheet in your own handwriting.
- You can only use a Ti-30x IIS calculator. Unless otherwise stated, you have to give exact answers to questions. ($\frac{2\ln 3}{\pi}$ and $1/3$ are exact, 0.699 and 0.333 are approximations for those numbers.)
- **Show your work**. If I cannot read or follow your work, I cannot grade it. **You may not get full credit for a right answer if your answer is not justified by your work.**

1. The parts of this question are not related.

   (a) (4 points) Evaluate $\lim_{x\to\infty} \dfrac{\sqrt{9+x}-3}{4-\sqrt{16+2x}}$ showing your steps.

   (b) (5 points) Use the *limit definition of the derivative* to find $f'(3)$ for $f(x)=\sqrt{2x+1}$. You will not get points if you differentiate using the Chain Rule.

   (c) (3 points) Evaluate the limit $\lim_{x\to\frac{\pi}{6}} \dfrac{\sin\left(\frac{\pi}{6}\right)-\sin x}{\frac{\pi}{6}-x}$ by viewing it as the definition of the derivative for some function $f(x)$. You will not get points if you use L'Hospital's Rule.

2. The parts of this question are not related.

(a) (5 points) Find $f'(x)$ if $f(x) = \dfrac{3e^x}{5} + \sqrt{4x} - \dfrac{3}{x^2} + x^e + 13x$.

(b) (4 points) If $g(x) = \dfrac{8x^6 + 7}{5 + 4\sqrt[3]{x}}$, what is $g'(1)$?

(c) (5 points) Find the values of A and B so that the parabola given by

$$y = Ax^2 + B$$

is tangent to the line $y = 5x - 4$ at the point where $x = 1$.

3. (11 points) Let

$$f(x) = \frac{x}{x^2 + x - 6}.$$

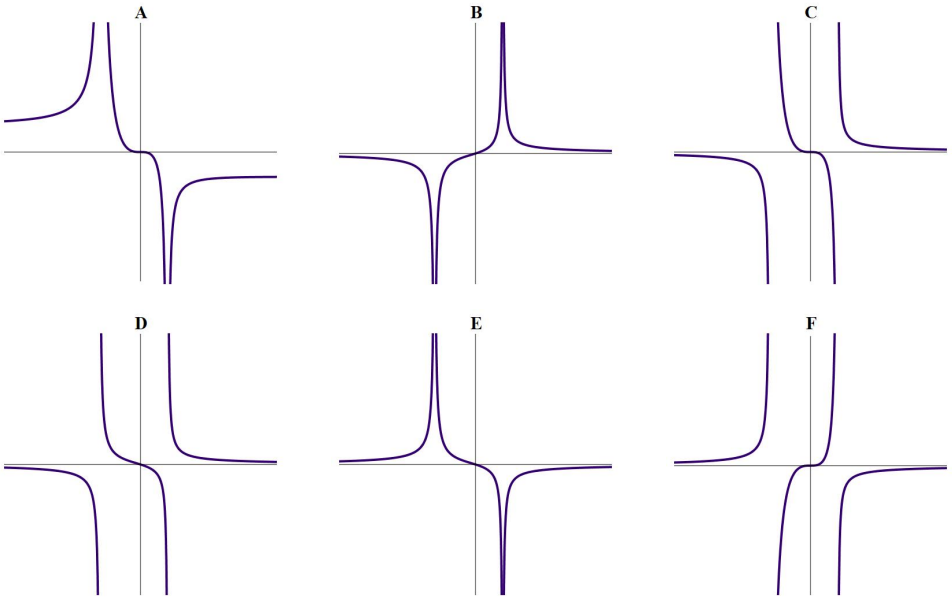
(a) Find the following limits. You do not have to show work here.

$\lim_{x \to 2^+} f(x) =$ $\lim_{x \to 2^-} f(x) =$ $\lim_{x \to \infty} f(x) =$ $\lim_{x \to -\infty} f(x) =$

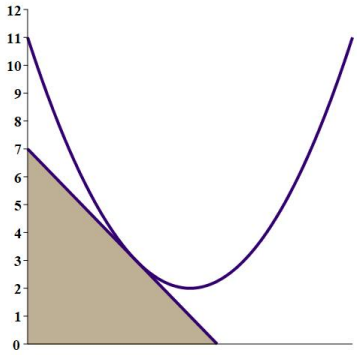
Also, find

$f'(x) =$ $f'(0) =$

(b) From your limits, you can see that the graph of the function $y = f(x)$ is given by picture ☐



4. (13 points) The line with y-intercept 7 is tangent to the parabola given by the equation $y = x^2 - 6x + 11$ as shown. Find the area of the shaded triangle.



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