In the below, $\mathbb{F} \in \{\mathbb{R}, \mathbb{C}\}$.

**P1 (15pts).**

a) What are the possible Jordan canonical forms for $A$ if $p_A(t) = (t - 2)^4(t + 1)^2$?

b) Find the Jordan canonical form for $A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.

**P2 (30pts).**

a) For a Jordan block $J_k(\lambda) = \begin{pmatrix} \lambda & 1 & \ldots & 0 \\ 0 & \lambda & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & \lambda & \lambda \end{pmatrix}$, calculate $(J_k(\lambda))^p$.

b) Show that if $A \in \mathcal{M}_n$ is nilpotent and $A_{i,i+1} \neq 0$ for any $i = 1, \ldots, n-1$, then $A$ is similar to $J_n(0)$. (Hint: could $A^2$ be 0? what happens to the “moving upper diagonal”?)

c) Show that if all eigenvalues of a matrix $A$ are 1, then $A$ is similar to $A^k$ for any $k > 0$ integer.

**P3 (15pts).** A matrix $A \in \mathcal{M}_n$ is *convergent* if all the entries of $A^m$ converge to 0 as $m \to \infty$. Find a necessary and sufficient condition for a matrix $A$ to be convergent.

**P4 (10pts).** Use the Schur form to show that if $A$ is normal, then $A$ is unitarily diagonalizable.