P1 (10pts). Show that there are at least 2 monochromatic triangles in any 2-color edge coloring of $K_6$. (In class, we showed there is at least 1.)

P2 (10pts). Consider the $n$-dimensional Euclidean space $\mathbb{R}^n$, and color every point in it using one of $n$ colors. Show that there must be two points of the same color at Euclidean distance 1.

P3 (10pts). For any $k \geq 2$ an integer, show that $R(3,3,\ldots,3)$ (with $k$ 3s) exists, and give a bound on how large it may be. This should be an explicit bound involving $k$, perhaps in terms of a sum; it should not be a recurrence. (Hint: it is easiest to do this thing inductively; in class, we showed that $R(3,3,3) \leq 17 = 3 \cdot (6 - 1) + 2$; think on how we proved it and how you can add one more color.)

P4 (10pts). By using the fact that for any integer $n \geq 2$,

$$2n^{n+\frac{1}{2}}e^{-n} \leq n! \leq 3n^{n+\frac{1}{2}}e^{-n}$$

(aka the Stirling formula) give a slightly tighter upper bound for $R(k, k)$ than $4^{k-1}$ (at least for $k$ large), using the fact that

$$R(k, k) \leq \binom{2(k-1)}{k-1}.$$

Bonus Problem (10pts). Do problem 2, but now with $n + 1$ colors allowed.