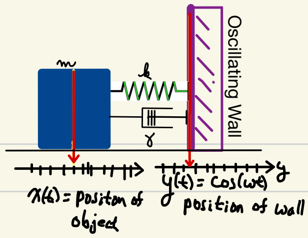


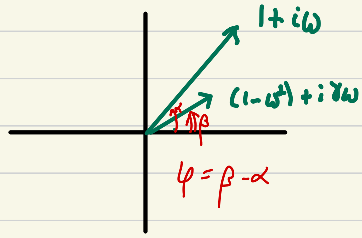
$$m x'' = -k(x - y(t)) - \gamma(x' - y'(t))$$

$$m x'' + \gamma x' + kx = k\,y(t) + \gamma y'(t)$$

$$k\cos(\omega t) - \gamma\omega \sin(\omega t)$$

Steady state solution.

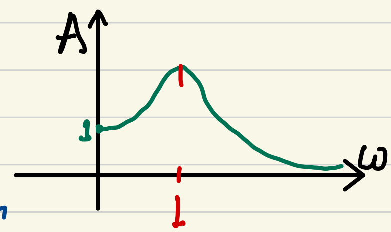
$$x'' + \gamma x' + x = \cos(\omega t) - \gamma\omega\sin(\omega t)$$



$$z'' + \gamma z' + z = (1 + i\gamma\omega)\, e^{i\omega t}$$

$$z = G e^{i\omega t} \;:\; \{(1-\omega^2) + \gamma\omega i\}\, G = 1 + i\gamma\omega$$

$$G = \frac{1 + i\gamma\omega}{(1-\omega^2) + \gamma\omega i} = A e^{-i\varphi} \qquad x = A\cos(\omega t - \varphi)$$



$$A = |G| = \sqrt{\frac{1 + (\gamma\omega)^2}{(1-\omega^2)^2 + (\gamma\omega)^2}} = \sqrt{\frac{1 + (\gamma\omega)^2}{1 + (\gamma^2 - 2)\omega^2 + \omega^4}}$$

$$\frac{1}{\gamma}\sqrt{\sqrt{2\gamma^2+1} - 1}$$

Review:

- 1st order
  - direction field
  - stable, unstable, semistable critical points
  - separable ODEs
  - integrating factors
  - Euler's method
- 2nd order ODE
  - Char. poly
  - undetermined coeffs.
  - resonance
  - steady state soln

Laplace transform
- computing $\mathcal{L}\{f(t)\}$
- computing $\mathcal{L}^{-1}\{F(s)\}$
- $u_a(t)$, $\delta_a(t)$
- convolution formula:

$$a x'' + b x' + c x = f(t)$$

$$x(0) = 0 \quad x'(0) = 0$$

$$x(t) = f * g\,(t)$$

$g(t)$ = unit impulse response.

Models:
- Law of Cooling
- Mixing
- mechanical systems