Math 565

Winter 2020

Homework 1

1. Let C_* and C'_* be chain complexes over the principal ideal domain R, and let $[C_*, C'_*]$ denote the chain homotopy classes of chain maps from C_* to C'_*. This becomes an R-module in the evident way: If $[f]$ and $[g]$ are chain maps, define $r[f] + [g] = [rf + g]$, where $[\ ]$ denotes the equivalence class in $[C_*, C'_*]$. There is then a homomorphism

$$\varphi : [C_*, C'_*] \to \mathrm{Hom}_R(H_*(C_*), H_*(C'_*))$$

defined by $\varphi([f]) = f_*$. (The R-module on the right is the R-module of graded R-module homomorphisms from $H_*(C_*)$ to $H_*(C'_*)$.)

a. If C_* is a free chain complex over R, prove that φ is an epimorphism.

b. If C_* is a free chain complex over R and $H_*(C_*)$ is also free over R, prove that φ is an isomorphism.