Math 308 Discussion Problems
Winter 2017

(1) Matt is a software engineer writing a script involving 6 tasks. Each must be done one after the other. Let t_i be the time for the ith task. These times have a certain structure:

•Any 3 adjacent tasks will take half as long as the next two tasks.

•The second task takes 1 second.

•The fourth task takes 10 seconds.

a) Write an augmented matrix for the system of equations describing the length of each task.
b) Reduce this augmented matrix to reduced echelon form.
c) Suppose he knows additionally that the sixth task takes 20 seconds and the first three tasks will run in 50 seconds. Write the extra rows that you would add to your answer in b) to take account of this new information.
d) Solve the system of equations in c).

(2) Find a 3×4 matrix A, in *reduced* echelon form, with free variable x_3, such that the general solution of the equation $A\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$ is

$$\mathbf{x} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 6 \end{bmatrix} + s \begin{bmatrix} -1 \\ 2 \\ 1 \\ 0 \end{bmatrix},$$

where s is any real number.

(3) Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

Find a 3×2 matrix B with $AB = I_2$. Is there more than one matrix B with this property? Justify your answer.

(4) Find all values z_1 and z_2 such that $(2, -1, 3)$, $(-4, z_1, z_2)$ and $(1, 2, 2)$ do not span $\mathbb{R}^3$.

(5) Find a 2×2 matrix A, which is not the zero or identity matrix, satisfying each of the following equations.
a) $A^2 = 0$
b) $A^2 = A$
c) $A^2 = I_2$

(6) Find an *invertible* $n \times n$ matrix A and an $n \times n$ matrix B such that $\operatorname{rank}(AB) \neq \operatorname{rank}(BA)$, or explain why such matrices cannot exist.

(7) Find a 3×2 matrix A and a 2×3 matrix B such that AB is invertible or explain why such matrices cannot exist. Answer the same question with the requirement that BA be invertible.

(8) Find a 3×4 matrix A with nullity 2 and with

$$\operatorname{col}(A) = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \right\},$$

or explain why such a matrix can't exist.

(9) Find a 3×3 matrix A and a 3×3 matrix B, each with nullity 1, such that AB is the 0 matrix, or explain why such matrices cannot exist.

(10) Find the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ -2 & 0 & 4 \end{bmatrix}^3 \begin{bmatrix} 8 & 0 & 3 \\ -1 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix}^{-1}.$$

(11) Let

$$B = \begin{bmatrix} 1 & z \\ 4 & 3 \end{bmatrix}.$$

Find all values of z such that the linear transformation T induced by B fixes no line in $\mathbb{R}^2$.

(12) Let S be a plane in $\mathbf{R}^3$ passing through the origin, so that S is a two-dimensional subspace of $\mathbf{R}^3$. Say that a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ is a *reflection about* S if $T(\mathbf{v}) = \mathbf{v}$ for any vector $\mathbf{v}$ in S and $T(\mathbf{n}) = -\mathbf{n}$ whenever $\mathbf{n}$ is perpendicular to S. Let T be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$\frac{1}{3}\begin{bmatrix} -1 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

This linear transformation is the reflection about a plane S. Find a basis for S.

(13) Suppose $S = \operatorname{span}\{\mathbf{u}_1, \mathbf{u}_2\}$, where $\mathbf{u}_1 = (1, 4, 6)$ and $\mathbf{u}_2 = (2, 1, -8)$. Let $\mathcal{B}_1$ denote the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for S, and let $\mathcal{B}_2$ denote a second basis $\{\mathbf{v}_1, \mathbf{v_2}\}$. If $[\mathbf{u}_1]_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}_{\mathcal{B}_2}$ and $[\mathbf{u}_2]_{\mathcal{B}_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{\mathcal{B}_2}$, find $\mathbf{v}_1$ and $\mathbf{v}_2$.

(14) Let S be the subspace of $\mathbb{R}^3$ spanned by the vectors $(1, 0, -1)$ and $(2, 1, 0)$. Find the 3×3 matrix A such that $A\mathbf{x} = \operatorname{proj}_S \mathbf{x}$ for all vectors $\mathbf{x}$.