Name ____________________________

Math 308E
Winter 2016

Final
March 16, 2016

Instructions

- Point totals for each problem are shown in parentheses.
- You must show all your work on the examination to receive credit. You must also use the techniques of this course on each problem. Ask if you are not sure about what is permitted.
- Read each problem carefully. You will not receive credit if you misunderstand or misread a problem.
- Your work must be neat and organized.
- Be very careful with your arithmetic. None of the calculations or answers are too complicated.
- Make sure your test has 8 questions.

(5) 1. For what value of c does the equation

$$\begin{bmatrix} 4 & 2 & c \\ 1 & 3 & -1 \\ 2 & 0 & 2 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 5 \\ 0 \\ 3 \end{bmatrix}$$

have an infinite number of solutions $\mathbf{x}$? With this value of c, find the general solution of the equation.

(5) 2. Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

Find a 3×2 matrix B with $AB = I_2$. Is there more than one matrix B with this property? Be sure to justify your answer.

(6) 3. Find a 2×2 matrix A, which is not the zero or identity matrix, satisfying each of the following equations.

a) $A^2 = 0$

b) $A^2 = A$

c) $A^2 = I_2$

(5) 4. Find the determinant of the matrix

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ -2 & 0 & 4 \end{bmatrix}^3 \begin{bmatrix} 8 & 0 & 3 \\ -1 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix}^{-1}.$$

(5) 5. Let S be a plane in $\mathbf{R}^3$ passing through the origin, so that S is a two-dimensional subspace of $\mathbf{R}^3$. Say that a linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^3$ is a *reflection about* S if $T(\mathbf{v}) = \mathbf{v}$ for any vector $\mathbf{v}$ in S and $T(\mathbf{n}) = -\mathbf{n}$ whenever $\mathbf{n}$ is orthogonal to S. Let T be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where A is the matrix

$$\frac{1}{3}\begin{bmatrix} -1 & -2 & 2 \\ -2 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}.$$

This linear transformation is the reflection about a plane S. Find a basis for S.

(5) 6. Suppose $S = \text{span}\{\mathbf{u}_1, \mathbf{u}_2\}$, where $\mathbf{u}_1 = (1, 4, 6)$ and $\mathbf{u}_2 = (2, 1, -8)$. Let $\mathcal{B}_1$ denote the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ for S, and let $\mathcal{B}_2$ denote a second basis $\{\mathbf{v}_1, \mathbf{v_2}\}$. If $[\mathbf{u}_1]_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}_{\mathcal{B}_2}$ and $[\mathbf{u}_2]_{\mathcal{B}_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{\mathcal{B}_2}$, find $\mathbf{v}_1$ and $\mathbf{v}_2$.

(5) 7. Let S be the subspace of $\mathbf{R}^4$ spanned by the vectors $(1, 0, 4, 2)$ and $(1, 2, 3, 5)$. Find a basis for $S^{\perp}$.

(5) 8. Let S be the subspace of $\mathbf{R}^3$ spanned by the vectors $(1, 0, -1)$ and $(2, 1, 0)$. Find the 3×3 matrix A such that $A\mathbf{x} = \operatorname{proj}_S \mathbf{x}$ for all vectors $\mathbf{x}$.