Math 308E
Spring 2016

Midterm 2
May 13, 2016

Instructions

- Point totals for each problem are shown in parentheses.

- You must show all your work on the examination to receive credit. You must also use the techniques of this course on each problem. Ask if you are not sure about what is permitted.

- Read each problem carefully. You will not receive credit if you misunderstand or misread a problem.

- Your work must be neat and organized.

- Be very careful with your arithmetic. None of the calculations or answers are too complicated.

- Make sure your test has 6 questions.

(4)   1. Find a, b, and c in the following matrix equation:

$$\begin{bmatrix} 5 & a \\ 4 & -5 \end{bmatrix} \begin{bmatrix} b & -4 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -25 \\ 9 & c \end{bmatrix}.$$

(5) 2. Let
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \end{bmatrix}.$$

Find a 3×2 matrix B with $AB = I_2$. Is there more than one matrix B with this property? Be sure to justify your answer.

(5)   3. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation with $T\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$ and

$T\left(\begin{bmatrix} 2 \\ -3 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -12 \end{bmatrix}$. Find $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$.

(5)   4. Let $S = \operatorname{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$, where $\mathbf{v}_1 = (2, 0, 2, 4)$, $\mathbf{v}_2 = (0, 2, -2, -2)$, $\mathbf{v}_3 = (1, 1, 0, 1)$, and $\mathbf{v}_4 = (4, 2, 2, -1)$. Find a basis for S which is a subset of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$. (Don't just write down an answer; you must show how you know that your answer is correct.)

(3) 5. Find a 2×3 matrix B with nullity 0 or explain why such a matrix can't exist.

(4) 6. Find a 3×4 matrix A with nullity 2 and with

$$\mathrm{col}(A) = \mathrm{span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \right\},$$

or explain why such a matrix can't exist.