NAME _____ TA'S NAME _____

STUDENT ID _____ SECTION _____

Math 124E, 124F
Winter 2011

Midterm 2
February 22, 2011

Solutions

Point totals are indicated in parentheses. You must show your work to receive credit.

(16) 1. Compute the derivative of the following functions. You need not simplify your answers.

a. $y = (1 + \sqrt{1 + x^2})^{1/3}$
b. $y = x^{(2^x)}$
c. $y = \ln(\sin^{-1}(\sqrt{x}))$
d. $y = \sin(x \cos(x^2))$

a. $y' = \frac{1}{3}(1+\sqrt{1+x^2})^{-2/3}\left(\frac{1}{2\sqrt{1+x^2}}\right)\cdot 2x$

$= \frac{1}{3}(1+\sqrt{1+x^2})^{-2/3}\left(\frac{x}{\sqrt{1+x^2}}\right)$

b. $\ln y = \ln x^{(2^x)} = 2^x \ln x$

$y'/y = 2^x \cdot \left(\frac{1}{x}\right) + 2^x \cdot \ln 2 \cdot \ln x$

$y' = x^{(2^x)}\left[2^x\left(\frac{1}{x} + \ln 2 \cdot \ln x\right)\right]$

c. $y' = \frac{1}{\sin^{-1}(\sqrt{x})} \cdot \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$

d. $y' = \cos(x\cos(x^2))\left[\cos(x^2) - x\sin(x^2)\cdot 2x\right]$

$= \cos(x\cos(x^2))\left[\cos(x^2) - 2x^2 \cdot \sin(x^2)\right]$

(6) 2. Find the slope of the line tangent to the curve

$$\cos(\pi xy) = x^2y - \frac{1}{2}$$

at the point $(-1, \frac{1}{2})$.

The slope of the tangent line is y' evaluated at the point with $x=-1$ and $y=\frac{1}{2}$.

Implicitly differentiate to get

$$-\sin(\pi xy)\left[\pi y + \pi x y'\right] = 2xy + x^2y'$$

Evaluate at $x=-1$ and $y=\frac{1}{2}$ to get

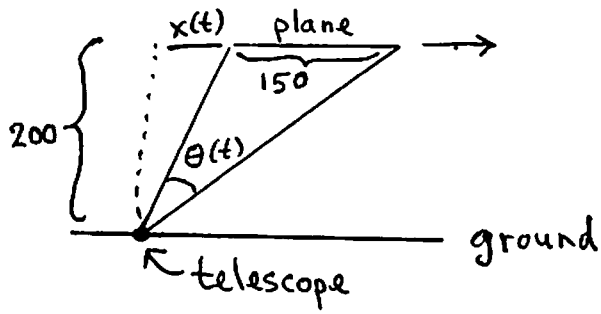
$$-\sin\left(-\frac{\pi}{2}\right)\left[\frac{\pi}{2} - \pi y'\right] = -1 + y'$$

$$\frac{\pi}{2} - \pi y' = -1 + y'$$

$$\frac{\pi}{2} + 1 = (1+\pi)y'$$

$$y' = \frac{\pi/2 + 1}{1+\pi} = \frac{\pi+2}{2+2\pi}$$

(10) 3. A low flying bombing plane, 150 feet long, is flying at an altitude of 200 feet. A telescope on the ground measures the angle $\theta(t)$ subtended by the plane at time t, as in the picture below. Suppose that this angle is changing at the rate of $-.9$ radians/sec at the instant that the tail of the plane is directly above the telescope. How fast is the plane flying at that time?



Let $x(t)$ = horizontal distance of the tail of the plane from the point directly above the telescope

We know: $\frac{d\theta}{dt} = -.9$ at time t when $x(t) = 0$

Want: $\frac{dx}{dt}$ at that time

$$\theta(t) = \tan^{-1}\left(\frac{150 + x(t)}{200}\right) - \tan^{-1}\left(\frac{x(t)}{200}\right)$$

$$\theta'(t) = \frac{1}{1 + \left(\frac{150 + x(t)}{200}\right)^2} \cdot \frac{x'(t)}{200} - \frac{1}{1 + \left(\frac{x(t)}{200}\right)^2} \cdot \frac{x'(t)}{200}$$

At time t when $x(t) = 0$:

$$-.9 = \frac{1}{1 + (3/4)^2} \cdot \frac{x'(t)}{200} - 1 \cdot \frac{x'(t)}{200}$$

$$-.9 = \left(\frac{16}{25} - 1\right) \frac{x'(t)}{200}$$

$$-\frac{9}{10} = \left(-\frac{9}{25}\right) \frac{x'(t)}{200}$$

$$x'(t) = 20 \cdot 25 = 500 \text{ ft./sec.}$$

(8) 4. Let c be a positive constant, and let C be the curve defined by the parametric equations

$$\begin{aligned} x(t) &= ct^2 + t - 2 \\ y(t) &= 2t^2 + ct - 8 \end{aligned}$$

for $t > 0$. The point $(4c, 2c)$ is the point on the curve corresponding to the value $t = 2$ of the parameter.

a. Find the equation of the line tangent to C at $(4c, 2c)$.

b. For what value of c does this tangent line pass through $(0, 0)$?

a. The slope of this line is $\frac{y'(2)}{x'(2)}$.

$y'(t) = 4t + c$

$x'(t) = 2ct + 1$

$\frac{y'(2)}{x'(2)} = \frac{8+c}{4c+1}$

equation of the tangent line:

$(y - 2c) = \frac{8+c}{4c+1}(x - 4c)$

b. tangent line passes through $(0,0)$ precisely when

$(0 - 2c) = \frac{8+c}{4c+1}(0 - 4c)$

$-2c = -4c\left(\frac{8+c}{4c+1}\right)$

$1 = \frac{16+2c}{4c+1}$

$4c + 1 = 16 + 2c$

$2c = 15$

$c = 15/2$