NAME Solutions TA'S NAME ____________

STUDENT ID ____________ SECTION ____________

Math 124C
Winter 2012

Midterm 1
January 31, 2012

Point totals are indicated in parentheses. You must show your work to receive credit. You do not need a calculator for any of the problems; consequently, you will not receive credit for any solution based on calculator computations.

(12) 1. Evaluate the following limits:

a. $\lim_{h\to 0}\left[\dfrac{\frac{1}{h^2+3h-1}+1}{h}\right]$

b. $\lim_{x\to\infty}\tan^{-1}\left(\dfrac{x-2x^3}{x^2+1}\right)$

$$\frac{\frac{1}{h^2+3h-1}+1}{h} = \frac{1+(h^2+3h-1)}{h(h^2+3h-1)}$$

$$= \frac{h^2+3h}{h(h^2+3h-1)}$$

$$= \frac{h+3}{h^2+3h-1}$$

$$\therefore \lim_{h\to 0}\frac{\frac{1}{h^2+3h-1}+1}{h} = \lim_{h\to 0}\frac{h+3}{h^2+3h-1}$$

$$= \frac{3}{-1} = -3$$

$$\frac{x-2x^3}{x^2+1} = \frac{x^2(\frac{1}{x}-2x)}{x^2(1+1/x^2)}$$

$$= \frac{\frac{1}{x}-2x}{1+\frac{1}{x^2}}$$

$$\therefore \lim_{x\to\infty}\frac{x-2x^3}{x^2+1} = \lim_{x\to\infty}\frac{\frac{1}{x}-2x}{1+1/x^2} = -\infty$$

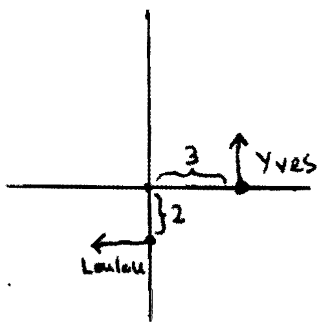
and so

$$\lim_{x\to\infty}\tan^{-1}\left(\frac{x-2x^3}{x^2+1}\right) = -\frac{\pi}{2}.$$

(12) 2. Yves is located 3 miles east of an intersection between two main roads. (See picture below.) He is walking north along a side street at the rate of 4 miles per hour. Loulou is located 2 miles south of this intersection and is walking west along another side street at the rate of 3 miles per hour.

a. Write a formula for the distance (in miles) between Yves and Loulou at time t (in hours).

b. What is the instantaneous rate of change of the distance between them at time $t = 0$? (You may not use any formulas for the derivative that you may have learned in a previous calculus course.)



a. $d(t) = \sqrt{(4t+2)^2 + (3t+3)^2}$

$= \sqrt{25t^2 + 34t + 13}$

b. instantaneous rate of change at $t=0$ $= d'(0)$.

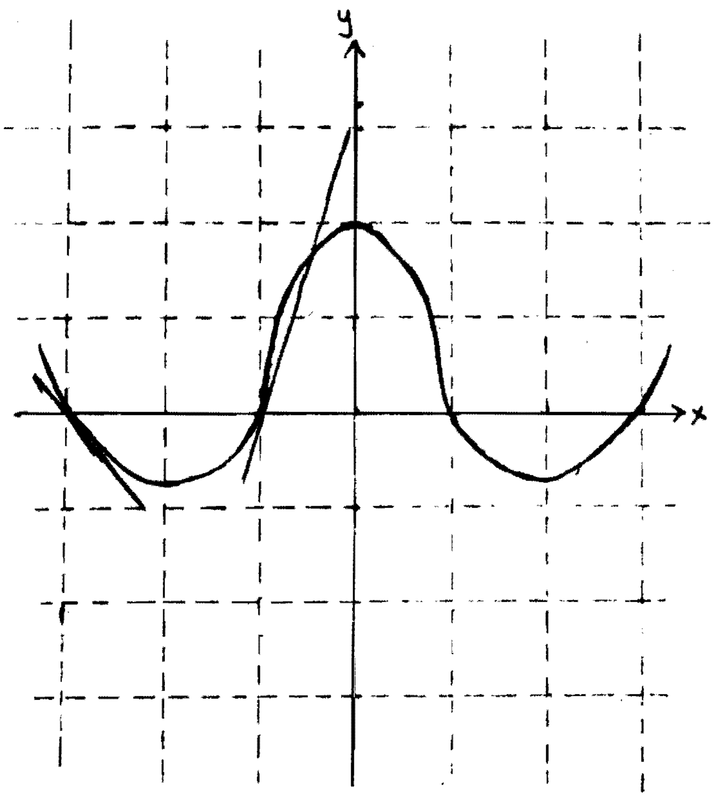
$$d'(0) = \lim_{h \to 0} \frac{\sqrt{25h^2+34h+13} - \sqrt{13}}{h}$$

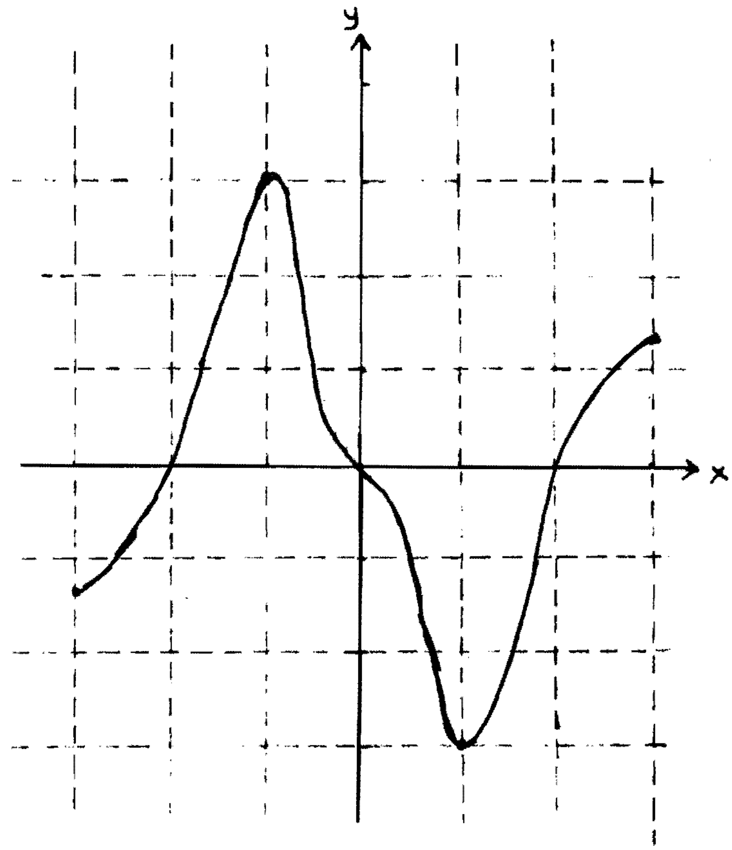
$$= \lim_{h \to 0} \frac{\left(\sqrt{25h^2+34h+13} - \sqrt{13}\right)}{h} \cdot \frac{\left(\sqrt{25h^2+34h+13} + \sqrt{13}\right)}{\left(\sqrt{25h^2+34h+13} + \sqrt{13}\right)}$$

$$= \lim_{h \to 0} \frac{(25h^2+34h+13) - 13}{h\left(\sqrt{25h^2+34h+13} + \sqrt{13}\right)}$$

$$= \lim_{h \to 0} \frac{25h+34}{\sqrt{25h^2+34h+13} + \sqrt{13}} = \frac{34}{2\sqrt{13}} = \frac{17}{\sqrt{13}} \text{ miles/hour.}$$

(10) 3. The graph of an even function f is shown below. Use this graph to estimate $f'(-3)$, $f'(-2)$, $f'(-1)$, $f'(0)$, $f'(1)$, $f'(2)$, and $f'(3)$. (If any of these derivatives don't exist, explain why.) Then sketch the graph of the derivative function f'.





$f'(0) = 0$

$f'(-3) \approx -4/3$

$f'(-2) = 0$

$f'(-1) \approx 3$

symmetry $\Rightarrow$ $f'(1) \approx -3$

$f'(2) = 0$

$f'(3) \approx 4/3$

(8) 4. Let $g(x)$ be the function defined by

$$g(x) = \begin{cases} 2(x-1) & x \le 0 \\ x^2 - 1 & x > 0. \end{cases}$$

(2) a. Find $(g \circ g)(1)$.

(6) b. Find $\lim_{h \to 0^-} \frac{(g \circ g)(1+h) - (g \circ g)(1)}{h}$. (You must clearly show how you obtain this limit—don't just write an answer.)

a. $(g \circ g)(1) = g(g(1)) = g(0) = -2$

b. If $h > -1$, $g(1+h) = (1+h)^2 - 1$, and if $-1 < h < 0$, $(1+h)^2 - 1 < 0$. Thus

$$(g \circ g)(1+h) = g(g(1+h)) = 2[(1+h)^2 - 1 - 1] = 2[h^2 + 2h - 1]$$

for $-1 < h < 0$, and

$$\lim_{h \to 0^-} \frac{(g \circ g)(1+h) - (g \circ g)(1)}{h} = \lim_{h \to 0^-} \frac{2[h^2 + 2h - 1] + 2}{h}$$

$$= \lim_{h \to 0^-} \frac{2h^2 + 4h}{h}$$

$$= \lim_{h \to 0^-} 2h + 4 = 4.$$

(12) 5. Suppose that a and b are constants and that the curve $y = ax^3 + x^2 + bx + 3$ passes through the point $(-1, 1)$. In addition, the line tangent to the curve at this point has equation $y = 5x + 6$. Find a and b.

Since the curve passes through $(-1,1)$, we have

$1 = -a + 1 - b + 3$, so $a + b = 3$

Since $y' = 3ax^2 + 2x + b$, the slope of the line tangent to the curve at $(-1,1)$ is $3a - 2 + b$. The line $y = 5x + 6$ has slope 5; therefore

$3a - 2 + b = 5$.

Putting the above two equations together, we have

$$a + b = 3$$
$$3a + b = 7$$

Solving this system of equations yields $a = 2$, $b = 1$.