Your Name

Your Signature

Student ID #

	Megan		David		Fablina		Sam	
Section (Tues.)	10:30	9:00	10:30	9:00	10:00	11:30	10:00	11:30
(circle one)	DA	DB	DC	DD	EA	EB	EC	ED

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK) but no worked problems. Do not share notes.
- You can use only a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 5 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	16	
2	16	
3	16	
4	16	
5	16	
Total	80	

1. Compute the derivatives of the following functions. You do not need to simplify your answers.

(a) (5 points) $f(x) = \sqrt{\cos^2 x + 5x^7}$

$$= (\cos^2 x + 5x^7)^{1/2}$$

$$f'(x) = \frac{1}{2}(\cos^2 x + 5x^7)^{-1/2}(2\cos(x)(-\sin(x)) + 35x^6)$$

$$= \frac{-2\cos(x)\sin(x) + 35x^6}{2\sqrt{\cos^2 x + 5x^7}}$$

(b) (5 points) $f(x) = \frac{1}{x} \cdot 3^{x^2-x}$

$$f'(x) = \frac{-1}{x^2} \cdot 3^{x^2-x} + \frac{1}{x} \cdot 3^{x^2-x} \cdot \ln(3) \cdot (2x-1)$$

(c) (6 points) $f(x) = (x + \arcsin(x))^{\ln x}$

$$\ln(f(x)) = \ln((x+\arcsin(x))^{\ln x})$$

$$\ln(f(x)) = \ln(x)\ln(x+\arcsin(x))$$

$$\frac{1}{f(x)} f'(x) = \frac{1}{x} \cdot \ln(x+\arcsin(x)) + \ln(x) \cdot \frac{1}{x+\arcsin(x)} \cdot \left(1 + \frac{1}{\sqrt{1-x^2}}\right)$$

$$f'(x) = (x+\arcsin(x))^{\ln x}\left[\frac{1}{x} \cdot \ln(x+\arcsin(x)) + \frac{\ln(x)}{x+\arcsin(x)}\left(1+\frac{1}{\sqrt{1-x^2}}\right)\right]$$

2. Consider the parametric curve
$$x = \cos t, \quad y = \sin t \cos t.$$

(a) (8 points) Find all values of t in the interval $[0, 2\pi]$ at which the tangent line to the curve is vertical.

$\frac{dx}{dt} = -\sin(t) = 0 \Rightarrow \boxed{t = 0, \pi, 2\pi}$

$\frac{dy}{dt} = -\sin^2(t) + \cos^2(t)$

vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$

$-\sin^2(0) + \cos^2(0) = 1 \neq 0$ ✓

$-\sin^2(\pi) + \cos^2(\pi) = 1 \neq 0$ ✓

$-\sin^2(2\pi) + \cos^2(2\pi) = 1 \neq 0$ ✓

(b) (8 points) Find the equations of all the tangent lines to the curve at the point $(0,0)$ for t in the interval $[0, 2\pi]$.

Set $x(t) = \cos(t) = 0 \Rightarrow t = \pi/2, 3\pi/2$

$y(t) = \sin(t)\cos(t) = 0 \Rightarrow t = 0, \pi/2, \pi, 3\pi/2, 2\pi$

$(x(t), y(t)) = (0,0)$ when $t = \pi/2$ and $t = 3\pi/2$

@ $t = \pi/2$: $\frac{\text{rise}}{\text{run}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin^2(\pi/2) + \cos^2(\pi/2)}{-\sin(\pi/2)} = \frac{-1+0}{-1} = 1 = m$

$y - y_1 = m(x - x_1) \Rightarrow \boxed{y = x}$

@ $t = \frac{3\pi}{2}$: $\frac{\text{rise}}{\text{run}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-\sin^2(3\pi/2) + \cos^2(3\pi/2)}{-\sin(3\pi/2)} = \frac{-(-1)^2 + 0}{-(-1)} = -1 = m$

$y - y_1 = m(x - x_1) \Rightarrow \boxed{y = -x}$

3. Consider the curve

$$y^2(y^2-4)=x^2(x^2-9)$$

(a) (8 points) Find an equation of the tangent line to the curve at the point $(3,-2)$.

Implicitly differentiate

$$\frac{d}{dx}\left(y^2(y^2-4)\right)=\frac{d}{dx}\left(x^2(x^2-9)\right)$$

$$2y\frac{dy}{dx}(y^2-4)+y^2\left(2y\frac{dy}{dx}\right)=2x(x^2-9)+x^2(2x)$$

Evaluate at $(3,-2)$

$$2(-2)\left.\frac{dy}{dx}\right|_{(3,-2)}\cancel{((-2)^2-4)}^{0}+(-2)^2\left(2(-2)\left.\frac{dy}{dx}\right|_{(3,-2)}\right)=2(3)\cancel{(3^2-9)}^{0}+3^2(2(3))$$

$$-16\left.\frac{dy}{dx}\right|_{(3,-2)}=54 \qquad \left.\frac{dy}{dx}\right|_{(3,-2)}=\frac{54}{-16}=-\frac{27}{8}$$

Slope: $-\frac{27}{8}$ Point $(3,-2)$

$$y-(-2)=-\frac{27}{8}(x-3)$$

$$\boxed{y=-2-\frac{27}{8}(x-3)}$$

(b) (4 points) Treating $y=f(x)$ as an implicit function satisfying $f(3)=-2$, what is the linearization of f at $a=3$? [**Hint**: Think about how this relates to the previous part]

The linearization of f at $a=3$ is the tangent line at $(3,f(3))=(3,-2)$.

$$\boxed{L(x)=-2-\frac{27}{8}(x-3)}$$

(c) (4 points) Use the linearization you found in the previous part to estimate the value of y that corresponds to $x=3.1$.

$$y=f(3.1)\approx L(3.1)=-2-\frac{27}{8}(3.1-3)$$

$$=-2-\frac{27}{8}(0.1)$$

$$=-2-\frac{27}{80}=-\frac{187}{80}=\boxed{-2.3375}$$

4. (a) (8 points) Find the linearization of the function $f(x) = \sqrt[3]{1+x}$ at the point $a = 0$.

We want to compute $L(x) \approx f'(a)(x-a) + f(a)$ where $a=0$

If $f(x) = (1+x)^{1/3}$ then $f'(x) = \frac{1}{3}(x+1)^{-2/3}$ and

furthermore, $f'(0) = \frac{1}{3}(0+1)^{-2/3} = \frac{1}{3}$

$f(0) = \sqrt[3]{1+0} = 1$

So $\boxed{L(x) \approx \frac{1}{3}(x-0)+1 = \frac{1}{3}x+1}$

(b) (8 points) Use the linearization from the previous part to approximate the number $\sqrt[3]{1.1}$.

We use $L(x)$ to approximate $\sqrt[3]{1.1}$.

Note that if $\sqrt[3]{x+1} = \sqrt[3]{1.1}$ then $x+1 = 1.1$ so

$\underline{x = 0.1}$

So we compute $L(0.1)$ giving us

$$\sqrt[3]{1.1} \approx L(0.1) = \frac{1}{3}(0.1)+1 = \frac{1}{3}\left(\frac{1}{10}\right)+1 = \frac{1}{30}+1 = \frac{31}{30}$$

$\approx 1.0\overline{3}$

5. (a) (8 points) A particle is moving around the ellipse $9x^2 + 25y^2 = 225$. Its position is given by $x(t) = 5\cos t$ and $y(t) = 3\sin t$. At what rate is the distance between the particle and the point $(0,1)$ changing when the particle is at the point $(-5,0)$?

Let $d(t)$ be the distance between the particle and the point $(0,1)$ at time t.

Then $d^2 = (x-0)^2 + (y-1)^2 = x^2 + (y-1)^2$

$$\frac{d}{dt} d^2 = \frac{d}{dt}\left[x^2 + (y-1)^2\right]$$

$$2dd' = 2xx' + 2(y-1)y'$$

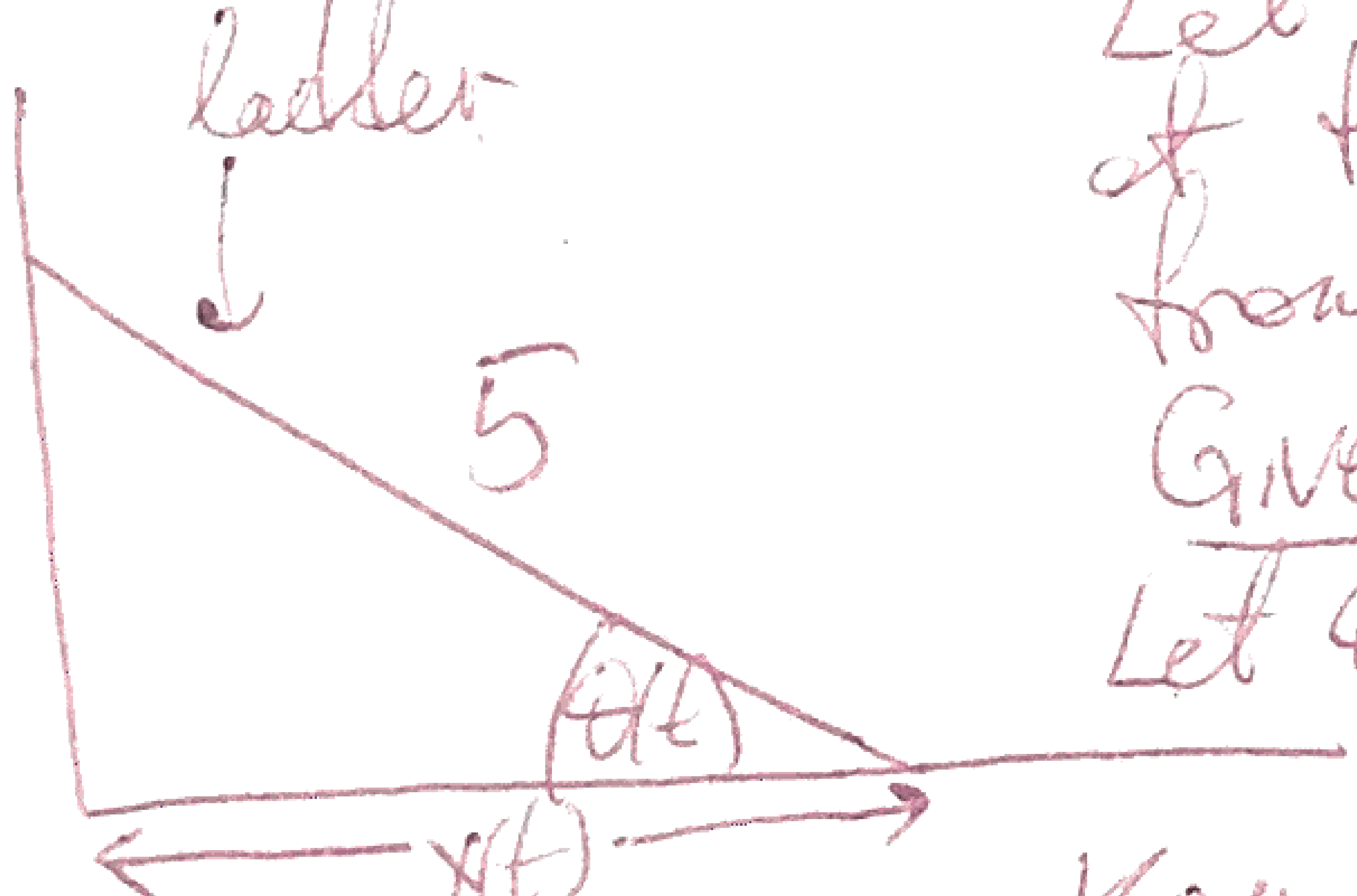
Plug in $(x,y) = (-5,0)$, $x'(t) = -5\sin t = -\frac{5}{3}(3\sin t) = -\frac{5}{3}y$

$d = \sqrt{5^2+1} = \sqrt{26}$ $\quad y'(t) = 3\cos t = \frac{3}{5}(5\cos t) = \frac{3}{5}x$

$$\Rightarrow 2\sqrt{26}\,d' = 2(-5)(0) + 2(-1)(-3) = 6$$

$$\Rightarrow d' = \frac{3}{\sqrt{26}}$$

(b) (8 points) A ladder 5 ft long is leaning against the side of a building. If the foot of the ladder is pulled away from the building at a constant rate of 1 foot per second, how fast is the angle formed by the ladder and the ground changing (in radians per second) at the instant when the top of the ladder is 3 feet from the ground?



Let $x(t)$ be the distance of the foot of the ladder from the wall at time t.

Given: $x' = 1$

Let $\theta(t)$ be the angle at time t.

Key Equation:

$$\cos\theta = \frac{x}{5} \Rightarrow 5\cos\theta = x$$

$$\frac{d}{dt}\left[5\cos\theta\right] = \frac{d}{dt}x$$

$$-5(\sin\theta)\theta' = x' \qquad \text{Plug in } x' = 1,\ \sin\theta = \frac{3}{5}$$

$$\Rightarrow \theta' = \frac{-x'}{5\sin\theta} \qquad \Rightarrow \theta' = -\frac{1}{3}$$