Lecture 23: Concavity.

Thm: Suppose f'' is continuous near c.

(a) If $f'(c)=0$ and $f''(c)>0$, then f has a local minimum at c.

(b) If $f'(c)=0$ and $f''(c)<0$, the f has a local maximum at c.

Note: If $f'(c)=0$ and $f''(c)=0$, then c may or may not be a minimizer/maximizer

Ex: Find the local maximum and minimum of f using both first and second derivative tests.

(a) $f(x)=1+3x^2-2x^3$

$f'(x)=6x-6x^2=6x(1-x)=0$

Crit points $x=0,\ x=1$

first derivative test:

	f'
$(-\infty,0)$	$-$
$(0,1)$	$+$
$(1,\infty)$	$-$

So $c=0$ is a local minimizer
$c=1$ is a local maximizer

Second derivative test: $f''(x)=6-12x$

$f''(0)=6>0$, $f''(1)=-6<0$

$\Rightarrow$ 0 local minimizer, 1 local maximizer

(b) $f(x) = \dfrac{x^2}{x-1}$

$$f'(x) = \frac{(x-1)2x - x^2(1)}{(x-1)^2} = \frac{2x^2-2x-x^2}{(x-1)^2} =$$

$$= \frac{x^2-2x}{(x-1)^2} = \frac{x(x-2)}{(x-1)^2}$$

So $f'(x) = 0 \iff x = 0,\ x = 2$

First Derivative test

	f'
$(-\infty, 0)$	$+$
$(0, 2)$	$-$
$(2, \infty)$	$+$

So $c = 0$ is a local maximizer
$c = 2$ is a local minimizer

Second Derivative Test:

$$f''(x) = \frac{(x-1)^2 \frac{d}{dx}(x^2-2x) - \left[\frac{d}{dx}(x-1)^2\right](x^2-2x)}{(x-1)^4}$$

$$= \frac{(x-1)^2(2x-2) - 2(x-1)(x^2-2x)}{(x-1)^4}$$

$$f''(0) = \frac{-2-0}{(-1)^4} = -2 < 0$$

$$f''(2) = \frac{(1)^2(2) - 2(1)(0)}{(1)^4} = 2 > 0$$

$\Rightarrow c = 0$ is a local maximizer. $c = 1$ is a local minimizer