Math 124: Lecture 1

1. Course Webpage
2. Course Webpage from the department
3. Circles and Tangent Lines

Equation of a line $L$ has equation $y = mx + b$. The slope $m = \frac{s-q}{r-p}$ with $b = y$-intercept when $x = 0$. If $L_1 \perp L_2$ then $m_1m_2 = -1$. The reason is $m_1m_2 = \tan \theta \cdot \tan \left( \frac{\pi}{2} + \theta \right)$.

Identifies: $\sin \left( \frac{\pi}{2} + \theta \right) = \cos \theta$ and $\cos \left( \frac{\pi}{2} + \theta \right) = -\sin \theta$.

Equation of a circle with center $(p, q)$ and radius $r$ is $r^2 = (x-p)^2 + (y-q)^2$.
Def: A line \( l \) is tangent to a circle if it intersects the circle only at a single point.

Slope of radial line: \( \frac{d-y}{x-a} \)

Slope of line \( l \): 

\[
\frac{-d-y}{x-a} = \frac{d-y}{x-a} 
\]

So the tangent line at \((a, d)\) is:

\[
y - d = -\left(\frac{a-x}{d-y}\right)(x-a) 
\]

**Question**: Consider the circle \( x^2 + y^2 = 25 \). What are the two tangent lines to the circle going through \((6, 5)\)?

Let tangent line be \( y = mx + b \) going through \((p, q)\) and \((6, 5)\):

\[
\frac{q}{p} = -\frac{5-q}{6-p} = -\frac{6-p}{5-q} 
\]

\[
5q - q^2 = -6p + p^2 
\]

\[
5q + 6p = 25 \quad p^2 + q^2 = 25 
\]

\[
q = 5 - \frac{6}{5}p 
\]

Plug into eqn of circle:

\[
p^2 + \left(5 - \frac{6}{5}p\right)^2 = 25 
\]

\[
p^2 + 25 + \frac{36}{25}p^2 - 12p = 25 
\]

\[
p(p + \frac{36}{25}p - 12) = 0 \Rightarrow p = 0, 52
\]

Compute \( q \) \(
\]

\[
q = \left(\frac{25}{61}\right) 
\]

Compute \( m \): \( p = -12 \left(\frac{25}{61}\right) \)