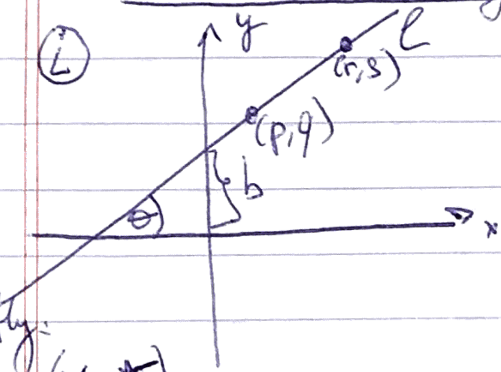
Math 124: Lecture 1

① Course Webpage
② Course Webpage from the department
③ Circles and Tangent Lines

(i)



Line ℓ has equ

$$y = mx + b$$

m = slope, b = y-intercept

$$m = \frac{s-q}{r-p} = \tan(\theta)$$

b = y-interscept = y value when $x=0$.

Equivalently:

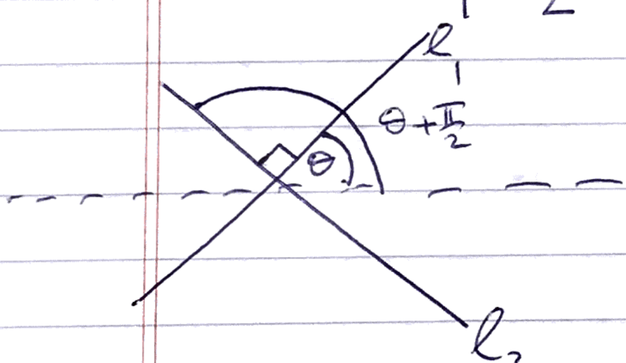
$y - s = m(\cancel{x - r})$

$y - q = m(x-p)$

(ii) If $\ell_1 \perp \ell_2$ with $\ell_1 : y = m_1 x + b_1$

$\ell_2 : y = m_2 x + b_2$

then $m_1 m_2 = -1$.



reason:

$$m_1 m_2 = \tan\theta \cdot \tan\left(\frac{\pi}{2}+\theta\right)$$

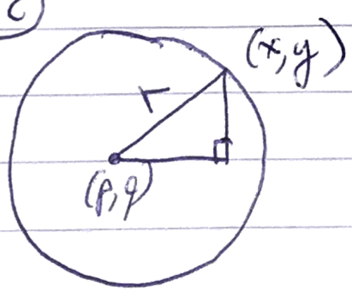
$$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\sin\left(\frac{\pi}{2}+\theta\right)}{\cos\left(\frac{\pi}{2}+\theta\right)}$$

$$= \frac{\sin\theta}{\cos\theta} \cdot \frac{\cos\theta}{-\sin\theta} = -1$$

Identities: $\sin\left(\frac{\pi}{2}+\theta\right) = \cos\theta$

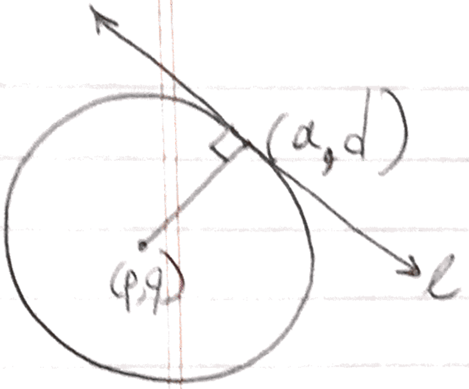
$\cos\left(\frac{\pi}{2}+\theta\right) = -\sin\theta$

(iii)



Equation of circle w/ center (p,q) and radius r

$$r^2 = (x-p)^2 + (y-q)^2$$



<u>Defn</u>: A line ℓ is <u>tangent</u> to a circle, if it intersects the circle only at a single point.

slope of radial line $= \dfrac{d-q}{a-p}$

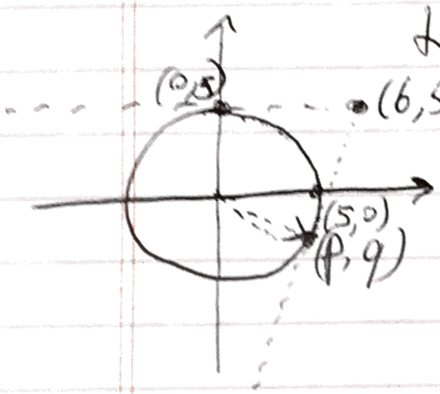
slope of line $\ell = -\dfrac{1}{\text{slope of rad line}}$

$= -\dfrac{a-p}{d-q}$

So the tangent line at (a,d) is

$$y-d = -\left(\frac{a-p}{d-q}\right)(x-a)$$

<u>Question</u>: Consider the circle $x^2+y^2=25$. What are the two tangent lines to the circle going through $(6,5)$?



<u>soln</u>: Let tangent line be $y=mx+b$ going through (p,q) and $(6,5)$

$$\Rightarrow \frac{q}{p} = -\frac{1}{\frac{5-q}{6-p}} = -\frac{6-p}{5-q}$$

$$\Rightarrow 5q - q^2 = -6p + p^2$$

$$\Rightarrow 5q + 6p = \cancel{25}\ p^2 + q^2 = 25$$

$$\Rightarrow q = 5 - \frac{6}{5}p$$

Plug into eqn of circle:

$$p^2 + \left(5 - \frac{6}{5}p\right)^2 = 25$$

$$p^2 + 25 + \frac{36}{25}p^2 - 12p = 25$$

$$p\left(p + \frac{36}{25}p - 12\right) = 0 \Rightarrow p = 0 \text{ or } p = -12\left(\frac{25}{61}\right)$$

Compute q $\Rightarrow$ compute m.