whose solution is $x_1 = \ln 2, x_2 = 0, x_3 = -\ln 2$. Therefore, the optimal solution of the primal problem is

$$t_1 = e^{x_1} = 2, \quad t_2 = e^{x_2} = 1, \quad t_3 = e^{x_3} = \frac{1}{2}.$$

Exercises

12.1. Find a dual problem to the convex problem

$$\begin{array}{ll} \min & x_1^2 + 0.5x_2^2 + x_1x_2 - 2x_1 - 3x_2 \\ \text{s.t.} & x_1 + x_2 \le 1. \end{array}$$

Find the optimal solutions of both the dual and primal problems.

12.2. Write a dual problem to the problem

$$\begin{array}{ll} \min & x_1 - 4x_2 + x_3^4 \\ \text{s.t.} & x_1 + x_2 + x_3^2 \le 2 \\ & x_1 \ge 0 \\ & x_2 \ge 0. \end{array}$$

Solve the dual problem.

12.3. Consider the problem

$$\begin{array}{ll} \min & x_1^2 + 2x_2^2 + 2x_1x_2 + x_1 - x_2 - x_3 \\ \text{s.t.} & x_1 + x_2 + x_3 \le 1 \\ & x_3 \le 1. \end{array}$$

(i) Is the problem convex?

(ii) Find an optimal solution of the problem.

(iii) Find a dual problem and solve it.

12.4. Consider the primal optimization problem

$$\begin{array}{ll} \min & x_1^4 - 2x_2^2 - x_2 \\ \text{s.t.} & x_1^2 + x_2^2 + x_2 \le 0. \end{array}$$

(i) Is the problem convex?

(ii) Does there exist an optimal solution to the problem?

(iii) Write a dual problem. Solve it.

(iv) Is the optimal value of the dual problem equal to the optimal value of the primal problem? Find the optimal solution of the primal problem.

12.5. Consider the problem

$$\begin{array}{ll} \min & 3x_1^2 + x_1x_2 + 2x_2^2 \\ \text{s.t.} & 3x_1^2 + x_1x_2 + 2x_2^2 + x_1 - x_2 \ge 1 \\ & x_1 \ge 2x_2. \end{array}$$

(i) Is the problem convex?

(ii) Find a dual problem. Is the dual problem convex?

Downloaded 05/31/19 to 205.175.118.138. Redistribution subject to SIAM license or copyright; see http://www.siam.org/journals/ojsa.php