Variational analysis with substructure

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Goals

- Intuitive notion of identifiable sets.
- Existence, calculus, properties.
- Connection to critical cones (Generalized Reduction Lemma).
- Illustration: Spectral functions.
- Generic existence (semi-algebraic setting).

Critical points

Definition (Generalized critical points)

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- For convex f, critical points are global minimizers.
- If f is $\mathbf{C}^1$-smooth, criticality reduces to $\nabla f(x) = 0$.

Motivation (Sensitivity Analysis)

Consider the perturbed functions

$$f_v(x) = f(x) - \langle v, x\rangle. \qquad [\textit{For simplicity}],$$

and suppose $\bar{x}$ is critical for $f_{\bar{v}}$, that is $\bar{v} \in \partial f(\bar{x})$.

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Sensitivity question: How do solutions x_v of

$$v \in \partial f(x),$$

vary, as we perturb v near $\bar{v}$?

Motivating example

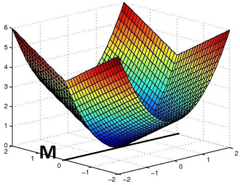

Figure: $f(x, y) = x^2 + |y|, \quad M = \{(t, 0) : -1 < t < 1\}$

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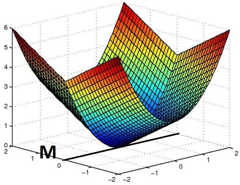

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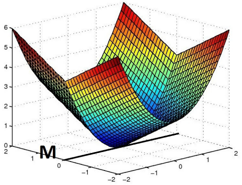

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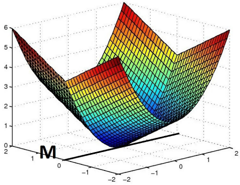

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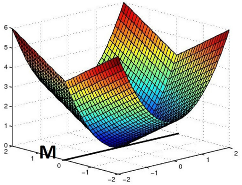

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All perturbed solutions x_v of $v \in \partial f(x)$ lie on $M \implies M$ captures all the sensitivity information!

- Only the restriction $f\big|_M$ matters!
- Goal: Look for small, well-behaved sets capturing only the essential information.

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$$v \in \partial f(x).$$

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Definition (Identifiable sets)

A set $M \subset \mathbf{R}^n$ is identifiable at $(\bar{x}, \bar{v}) \in \operatorname{gph} \partial f$ if locally near $\bar{x}$ have

$$M = \pi_{\mathbf{R}^n}((U \times V) \cap \operatorname{gph} \partial f),$$

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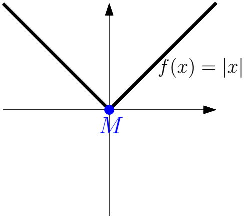

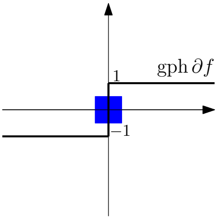

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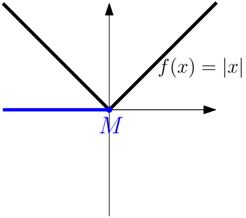

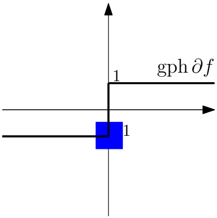

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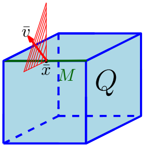

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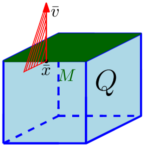

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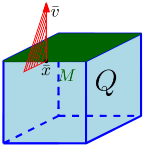

In this case $M = \bar{x} + K_Q(\bar{x}, \bar{v})$.

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Suppose M is an identifiable set at $(\bar{x}, 0) \in \operatorname{gph} \partial f$.

- *$\bar{x}$ is a (strict) local minimizer of $f \iff \bar{x}$ is a (strict) local minimizer of f on M.*
- *f grows quadratically near $\bar{x} \iff f$ grows quadratically on M near $\bar{x}$.*

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Definition

An identifiable set M at $(\bar{x}, \bar{v}) \in \operatorname{gph} \partial f$ is locally minimal if

$$M' \text{ identifiable at } (\bar{x}, \bar{v}) \Longrightarrow M \subset M', \text{ locally near } \bar{x}.$$

Existence and calculus

Locally minimal identifiable sets exist for

- fully amenable functions: $f(x) = g(F(x))$ where
  1. F is $\mathbf{C}^2$-smooth,
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A strong chain rule is available for composite functions

$$f(x) = g(F(x)).$$

Dimension Reduction

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Proposition (Reduction Lemma due to Robinson)

If Q is polyhedral, then

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Let M be a (prox-regular) identifiable set at $(\bar{x},\bar{v}) \in \mathrm{gph}\, N_Q(\bar{x})$. Then

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May use this to study nonpolyhedral variational inequalities!

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- When an identifiable manifold exists, nonsmoothness is not intrinsic...So can reduce analysis to the classical setting.

Lifts of identifiable manifolds

Consider $\mathbf{S}^n := \{n \times n \text{ symmetric matrices}\}$ and the eigenvalue map

$$A \mapsto (\lambda_1(A), \ldots, \lambda_n(A)),$$

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Identifiable manifolds "lift": (D, Lewis), (Daniilidis, Malick, Sendov)

$$M \text{ identifiable manifold at } (\bar{x}, \bar{v}) \in \operatorname{gph} \partial f$$
$$\Longrightarrow \lambda^{-1}(M) \text{ identifiable manifold at } (\bar{X}, \bar{V}) \in \operatorname{gph} \partial (f \circ \lambda).$$

Generic Properties

History: Rockafellar-Spingarn '79, considered problems

$$P(v,u): \quad \min \; f(x) - \langle v, x \rangle, \quad \text{s.t.} \; g_i(x) \le u_i, \text{ for all } i \in I := \{1, \ldots, m\},$$

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Theorem (Rockafellar-Spingarn '79)

- *For almost all (v,u), at every minimizer of $P(v,u)$:*

  Active manifold: active gradients are independent

  Strict complementarity: multipliers are strictly positive and

  Quadratic growth: objective function grows quadratically.

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Active manifold: existence of an identifiable manifold

Strict complementarity: $0 \in \operatorname{ri} \hat{\partial} f_v(x_v)$

Quadratic growth: f_v grows quadratically near x_v.

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Further, if $\dim \operatorname{gph} F = n = m$, then strong metric regularity is typical. Strong metric regularity of ∂f (i.e. tilt-stability) is equivalent to a uniform quadratic growth condition (D, Lewis '12).

Summary

- Presented the intuitive notion of identifiable sets.
- Showed how identifiable sets capture the essence of previously developed concepts (dimension reduction, critical cones, optimality conditions).
- Illustration: spectral functions.
- Generic properties of semi-algebraic optimization problems.

Thank you.