

MATH 3800: Theory of Numbers

Vanderbilt University, Fall 2026

LECTURE: SC 14??, TTh 1:15 – 2:30 pm

OFFICE HOURS: SC 1222, T 2:30 – 3:30 pm, W 1:30 – 2:30 pm

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*Chebyshev said it, and I'll say it again,
There's always a prime between n and $2n$.*

–Paul Erdős (folklore)

COURSE OVERVIEW. This course is an introduction to number theory, one of the oldest branches of mathematics. Historically, it was also considered to be (and lauded as!) one of the purest, precisely because it seemed bereft of outside applications; in an ironic twist, number theory today underpins much of modern cryptography and internet security, besides having deep connections to other areas of mathematics and beyond.

In this class, we will study core topics of elementary number theory, exploring properties of integers and prime numbers and how they behave under basic arithmetic operations. As we shall see, the simplicity of these operations belies complex and intricate phenomena; one cannot search far before stumbling upon some ancient, unsolved problem. The topics we shall consider include the Fundamental Theorem of Arithmetic, congruences, multiplicative functions, Fermat's Little Theorem, quadratic residues, Diophantine equations, Pythagorean triples, and primitive roots. Time permitting, there are various special topics we may explore, such as elliptic curves (and their cryptographic applications) or special cases of Fermat's Last Theorem.

PREREQUISITES. MATH 2100, 2501, or 2600. If you have any concerns about your preparedness for the course, please come talk to me.

TEXTBOOK. *Elementary Number Theory*, by James K. Strayer. 2002 reissue (ISBN: 9781577662242).

HOMEWORK. It is exceptionally difficult to learn mathematics without getting one's hands dirty. To that end, homework will be assigned on Brightspace on a weekly basis and will be due in class on Thursdays. In general, a proper subset of the problems will be graded, but it is important to make a serious attempt on all of them. Each topic we will discuss builds on the previous one, so it is hard to catch up once one falls behind. Accordingly, homework extensions will be granted only in extenuating circumstances. To account for this, the lowest homework grade will be dropped.

Homework submissions should be neatly written, with sufficient justification provided for each step. As a good rule of thumb, try to provide enough detail so that any of your classmates would be able to understand your argument (without reading your mind!). I strongly encourage the use of $\LaTeX$ for writing up your homework, and will provide a template which you are free to use and modify as needed. Exceptionally well-written homework assignments will receive a 5% bonus (up to a maximum of 105%). While handwritten work can earn this bonus, I find that the flexibility of using $\LaTeX$ is more conducive to producing clean, well-thought-out arguments.

CLASS PARTICIPATION. On certain days, we will break up the lecture with small group work or other activities. In particular, on days when homework is collected, we may spend some time going over solutions, and you may be called on to present at the board. This is not meant to quiz you, but instead to give you a chance to practice explaining math to others. Do not be afraid to make mistakes—mistakes are essential for learning anything!

While we are going over solutions together, you are welcome to correct any mistakes in your work, provided that you indicate clearly that you made changes and provided that the changes are in your own words (i.e., not copied verbatim from the board).

COLLABORATION AND TECHNOLOGY POLICY. You are encouraged to discuss the homework with your classmates, but you must write up your solutions independently. Please note next to each problem the names of your classmates with whom you collaborated. No outside sources beyond course materials may be used unless otherwise indicated. In particular, **generative AI may be used only for non-mathematical purposes**. (While it can be used for L^AT_EX formatting questions, it should not be used to transcribe handwritten work.) If you have any doubts about a particular use case for AI (or another tool), please err on the side of caution and ask me first.

EXAMS. There will be two midterm exams and a final exam on the following dates:

- Midterm 1: Oct 1 (tentatively)
- Midterm 2: Nov 12 (tentatively)
- Final Exam: Friday, December 18, 9:00 – 11:00 am

Notes, calculators, and electronic devices of any kind (phone, smartwatch, earbuds, etc.) are not permitted to be used during exams. Makeup exams will not be offered under normal circumstances; if you anticipate an unavoidable conflict with one or more exams, **please contact me as soon as possible** so that we can make alternative arrangements.

GRADES. Your raw grade in the course will be computed using the following distribution.

Class Participation	5%
Homework	30%
Midterm 1	20%
Midterm 2	20%
Final Exam	25%

Final letter-grade cutoffs will be determined at the end of the semester, but a raw grade of 65% is guaranteed to earn at least a passing grade, while 90% is guaranteed to earn at least an A-.

ACADEMIC INTEGRITY. Your work in this class is governed by Vanderbilt University's Honor Code. The work you submit for homework and exams must be your own. If you are uncertain about how the Honor Code applies, please ask me for clarification.

STUDENT ACCESS. If you believe you may require special accommodations for a condition that may impact your work in this course, please contact the Student Access Office as early as possible. The SAO will determine which accommodations are appropriate and communicate them to me. This service is confidential.