Math 3800: Theory of Numbers
Homework #1

YOUR NAME

Due September 3, 2026

- These problems rely on material from our upcoming lecture on Tuesday. Because this problem set has such a quick turnaround, I have tried to limit it to something that I think is doable in 1 to 3 hours.
- While the problems are more or less taken from Strayer, small changes have been made here and there—please keep that in mind if you refer back to the text! I have included some hints at the bottom of the second page, but I encourage you to refer to them only if you become well and truly stuck on a problem, or to check your work.
- Feel free to use the accompanying .TEX file as a homework template for this assignment. (There is a proof environment commented out below each problem statement.)
- Remember, every computation or claim should be sufficiently justified for full credit, and if you work with anyone else, please write their name(s) next to the problem.

§1.1 #3. Find the unique quotient q and remainder r guaranteed by the division algorithm for each dividend a and divisor b below:

(a) $a = 47$, $b = 6$

(b) $a = 281$, $b = 13$

(c) $a = 343$, $b = 49$

(d) $a = -105$, $b = 10$

(e) $a = -469$, $b = 31$

(f) $a = -500$, $b = 28$

§1.1 #4. If $a, b \in \mathbb{Z}$, find a necessary and sufficient condition that $a \mid b$ and $b \mid a$.

§1.1 #8. Let $a, n \in \mathbb{Z}_{>0}$. Prove that $a \mid (a+1)^n - 1$.

§1.1 #10. Let $n \in \mathbb{Z}$.

(a) Prove that $3 \mid n^3 - n$.

(b) Prove that $5 \mid n^5 - n$.

(c) Is it true that $4 \mid n^4 - n$? Provide a proof or counterexample.

The following excersise formalizes the calculation that we did at the end of the first class, where we reduced the Diophantine equation $x^4 - 117x + 31 = 0$ "modulo 2" (meaning that we forgot all information about the supposed integral solution x and the coefficients of the polynomial except whether they were even or odd).

§1.1 #11. Recall that $n \in \mathbb{Z}$ is called *even* if $2 \mid n$, and *odd* otherwise.

(a) Prove that n is even if and only if $n = 2m$ for some $m \in \mathbb{Z}$.

(b) Prove that n is odd if and only if $n = 2m + 1$ for some $m \in \mathbb{Z}$.

(c) Prove that the sum and product of two even integers are both even.

(d) Prove that the sum of two odd integers is even and that their product is odd.

(e) Prove that the sum of an even integer and an odd integer is odd and that their product is even.

§1.1 #12. Prove that the square of any odd integer is expressible in the form $8n + 1$ with $n \in \mathbb{Z}$.

Hints:

- #3(a): $q = 7, r = 5$, (c): $q = 7, r = 0$, (e): $q = -16, r = 27$
- #4: $a = \pm b$
- #8: Use the binomial theorem to expand $(a + 1)^n$, or prove it by induction on n.
- #10(a): Use induction.
- #12: Use #11(b).