Math 301 A - Spring 2014
Midterm Exam
April 23, 2014

Name: ______________________ Student ID no. : ______________

Signature: ______________________________________

1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
Total	60	

- Complete all 6 questions.
- Show all work for full credit.
- You have 50 minutes to complete the exam.

1. Find the gcd of $a = 163438$ and $b = 16150$ and express the gcd as a linear combination of a and b.

2. How many zeros does $321!$ end in (when written in decimal notation)?

3. (a) Give an example of two different positive integers with (exactly) 600 divisors.

(b) What is the smallest positive integer with (exactly) 600 divisors that you can find? (You do not have to find the smallest integer with 600 divisors, just try to find one as small as you can - the smaller the better!)

4. For what integers n is $n^2 - 1$ a prime number? State and prove a theorem that answers this question.

5. Prove that the product of two consecutive integers, both greater than 2, has at least three (not necessarily distinct) prime factors.

6. Prove that $\dfrac{\ln 2}{\ln 3}$ is irrational.

Here is a list of theorems and other facts that you can use without justification during the exam. This is only a partial list! Many minor results may be used without justification. This list is merely a reference of the more powerful and, perhaps, harder to remember ones.

- There are infinitely many primes.
- The transitivity of divisibilty: if a divides b, and b divides c, then a divides c.
- If d divides a and d divides b, then d divides any linear combination of a and b.
- If d and n are positive integers, and d divides n, then $d \leq n$.
- If a prime p divides ab, then p divides a or p divides b.
- The Fundamental Theorem of Arithmetic: all positive integers can be written in a unique way as a product of primes.
- The Euclidean algorithm for finding the gcd of two integers
- Stark, Theorem 2.3: n is a common divisor of a and b iff n divides $\gcd(a, b)$.
- Stark, Theorem 2.6: If $(n, a) = 1$ and $n|ab$, then $n|b$.
- Stark, Theorem 2.13: The n-th root of a positive integer is rational iff it is an integer.
- The result from problem 6 in the week 2 homework (i.e., if $d|n$, then the prime factorization of d consists only of primes from the prime factorization of n, with exponents no greater than the corresponding exponents in the prime factorization of n).
- The formulas for the number of, and sum of, divisors of a positive integer.
- For positive integers a and b, and any prime p,

$$\text{ord}_p ab = \text{ord}_p a + \text{ord}_p b \text{ and } \text{ord}_p(a + b) \leq \min\{\text{ord}_p a, \text{ord}_p b\}$$