

Math 300 B - Winter 2015  
Midterm Exam Number One  
January 28, 2015

Name: \_\_\_\_\_

Student ID no. : \_\_\_\_\_

Signature: \_\_\_\_\_

|       |    |  |
|-------|----|--|
| 1     | 10 |  |
| 2     | 10 |  |
| 3     | 10 |  |
| 4     | 10 |  |
| 5     | 9  |  |
| Total | 49 |  |

- Complete all 5 questions.
- You have 50 minutes to complete the exam.

1. Use a truth table to decide whether the following argument is valid. Explain your conclusion.

- Andie or Bing will sing.
- Bing and Clara will not both sing.
- Therefore, Andie and Bing will not both sing.

2. Let  $A, B$ , and  $C$  be sets. Verify the following identities by showing that the statement  $x \in$  "the left hand set" is equivalent to the statement  $x \in$  "the right hand set". Give a justification for each step (e.g., definition of union, DeMorgan's law, distributive law, etc.)

(a)  $A \cap B = A \setminus (A \setminus B)$

(b)  $(A \setminus B) \cup (B \setminus C) \cup (B \cap C) = A \cup B$

3. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

(a)  $P \wedge (Q \vee (P \wedge \neg Q))$

(b)  $(P \vee Q) \wedge (\neg P \wedge (\neg Q \vee P))$

4. Simplify the following expressions as much as possible. You should show a sequence of equivalent expressions connecting the original expression with your final one, and give a justification for each step (e.g., DeMorgan's law, distributive law, etc.)

(a)  $(\neg(P \wedge \neg Q) \rightarrow \neg P) \wedge Q$

(b)  $(Q \wedge P) \Leftrightarrow (Q \rightarrow P)$

5. Write useful contrapositives of the following sentences. Express the contrapositives as sentences, not symbolically.

(a) If  $n$  is prime, then  $n$  is odd or  $n = 2$ .

(b) If I am hungry or cold, then I am sad and afraid.

(c) If  $x$  is not zero, then there exists a number  $y$  such that  $xy > 1$ .

### DeMorgan's laws

$\neg(P \wedge Q)$  is equivalent to  $\neg P \vee \neg Q$

$\neg(P \vee Q)$  is equivalent to  $\neg P \wedge \neg Q$

### Commutative Laws

$P \wedge Q$  is equivalent to  $Q \wedge P$

$P \vee Q$  is equivalent to  $Q \vee P$

### Associative Laws

$P \wedge (Q \wedge R)$  is equivalent to  $(P \wedge Q) \wedge R$

$P \vee (Q \vee R)$  is equivalent to  $(P \vee Q) \vee R$

### Idempotent Laws

$P \wedge P$  is equivalent to  $P$

$P \vee P$  is equivalent to  $P$

### Distributive Laws

$P \wedge (Q \vee R)$  is equivalent to  $(P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R)$  is equivalent to  $(P \vee Q) \wedge (P \vee R)$

### Absorption Laws

$P \vee (P \wedge Q)$  is equivalent to  $P$

$P \wedge (P \vee Q)$  is equivalent to  $P$

### Double Negation Law

$\neg\neg P$  is equivalent to  $P$

### Tautology Laws

$P \wedge (\text{a tautology})$  is equivalent to  $P$

$P \vee (\text{a tautology})$  is a tautology

$\neg(\text{a tautology})$  is a contradiction

### Contradiction Laws

$P \wedge (\text{a contradiction})$  is a contradiction

$P \vee (\text{a contradiction})$  is equivalent to  $P$

$\neg(\text{a contradiction})$  is a tautology

### Conditional Laws

$P \rightarrow Q$  is equivalent to  $\neg P \vee Q$

$P \rightarrow Q$  is equivalent to  $\neg(P \wedge \neg Q)$

### Contrapositive Laws

$P \rightarrow Q$  is equivalent to  $\neg Q \rightarrow \neg P$

### Quantifier Negation Laws

$\neg\exists x P(x)$  is equivalent to  $\forall x \neg P(x)$

$\neg\forall x P(x)$  is equivalent to  $\exists x \neg P(x)$

### Sets

$A = B \Leftrightarrow ((x \in A) \Leftrightarrow (x \in B))$

$A \subseteq B \Leftrightarrow ((x \in A) \rightarrow (x \in B))$

$x \in A \cup B \Leftrightarrow ((x \in A) \vee (x \in B))$

$x \in A \cap B \Leftrightarrow ((x \in A) \wedge (x \in B))$

$x \in A \setminus B \Leftrightarrow (x \in A) \wedge (x \notin B)$