1. Let \( A \) be the statement \((P \lor (Q \land P)) \land (Q \lor P)\).

(a) Write out the truth table for \( A \).

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<tbody>
<tr>
<td>( P )</td>
<td>( Q )</td>
<td>( Q \land P )</td>
<td>( P \lor (Q \land P) )</td>
<td>( Q \lor P )</td>
<td>( (P \lor (Q \land P)) \land (Q \lor P) )</td>
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(b) Write a simpler expression which is equivalent to \( A \).

We can see from the table that \((P \lor (Q \land P)) \land (Q \lor P)\) is equivalent to \(P\).

2. (a) Show that, for sets \( A, B \) and \( C \),

\[(A \setminus B) \cap C = (A \cap C) \setminus B\]

using logical symbols and equivalences.

\[
x \in (A \setminus B) \cap C \\
\iff x \in A \land x \notin B \land x \in C \\
\iff (x \in A \land x \in C) \land x \notin B \\
\iff (x \in A \cap C) \land x \notin B \\
\iff x \in (A \cap C) \setminus B.
\]

Hence, \((A \setminus B) \cap C = (A \cap C) \setminus B\).

(b) Show that, for sets \( A, B \) and \( C \),

\[(B \cup C) \setminus (A \setminus B) = B \cup (C \setminus A)\]

using logical symbols and equivalences.

\[
x \in (B \cup C) \setminus (A \setminus B) \\
\iff (x \in B \lor x \in C) \land x \notin (A \setminus B) \\
\iff (x \in B \lor x \in C) \land \lnot(x \in A \land x \notin B) \\
\iff (x \in B \lor x \in C) \land (x \notin A \lor x \in B) \\
\iff (x \in B) \lor (x \in C \land x \notin A) \\
\iff x \in B \cup (C \setminus A).
\]

Hence, \((B \cup C) \setminus (A \setminus B) = B \cup (C \setminus A)\).
3. Write a useful negation of each of the following statements using idiomatic mathematical English.

(a) *For every real number* \( x \), *there exists a real number* \( y \) *which is greater.*
   
   There exists a real number \( x \) such that, for every real number \( y \), \( y \leq x \).

(b) *Every integer greater than 1 is either prime or composite.*
   
   There exists an integer greater than 1 which is not prime and not composite.

(c) *There exists a unique integer* \( n \) *which is divisible by 5 and 7.*
   
   Either there exists no integer which is divisible by 5 and 7 or there exists more than one integer which is divisible by 5 and 7.

(d) *Some functions are differentiable.*
   
   All functions are not differentiable.

4. Find a formula involving only the connectives \( \neg \) and \( \to \) that is equivalent to

\[
(P \lor Q) \land \neg(P \land Q)
\]

There are many correct answers. One is

\[
\neg((\neg P \to Q) \to \neg(P \to \neg Q))
\]

which can be arrived at like this:

\[
(P \lor Q) \land \neg(P \land Q) \\
\Leftrightarrow (P \lor Q) \land (\neg P \lor \neg Q) \\
\Leftrightarrow \neg((P \lor Q) \lor (\neg P \lor \neg Q)) \\
\Leftrightarrow \neg(((P \lor Q) \to \neg(P \lor \neg Q)) \\
\Leftrightarrow \neg((\neg P \to Q) \to \neg(P \to \neg Q))
\]

5. Simplify the expression below as much as you can.

\[
\neg(\neg P \land Q) \lor (P \land \neg R)
\]

This expression can be simplified to \( P \lor \neg Q \), or, equivalently, \( Q \to P \).