

Math 300 A - Winter 2013

Midterm Exam

January 30, 2013

I usually solve problems by letting them devour me.
- Franz Kafka, in a letter to Max Brod

Name: _____

Student ID no. : _____

Signature: _____

Section: _____

1	10	
2	10	
3	8	
4	10	
5	10	
Total	48	

- Complete all five questions.
- You have 50 minutes to complete the exam.

1. Let A be the statement $(P \vee (Q \wedge P)) \wedge (Q \vee P)$.

(a) Write out the truth table for A .

(b) Write a simpler expression which is equivalent to A .

2. (a) Show that, for sets A, B and C ,

$$(A \setminus B) \cap C = (A \cap C) \setminus B$$

using logical symbols and equivalences.

- (b) Show that, for sets A, B and C ,

$$(B \cup C) \setminus (A \setminus B) = B \cup (C \setminus A)$$

using logical symbols and equivalences.

3. Write a useful negation of each of the following statements using idiomatic mathematical English.

(a) For every real number x , there exists a real number y which is greater.

(b) Every integer greater than 1 is either prime or composite.

(c) There exists a unique integer n which is divisible by 5 and 7.

(d) Some functions are differentiable.

4. Find a formula involving only the connectives $\neg$ and $\rightarrow$ that is equivalent to

$$(P \vee Q) \wedge \neg(P \wedge Q).$$

5. Simplify the expression below as much as you can.

$$\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$$

DeMorgan's laws

$\neg(P \wedge Q)$ is equivalent to $\neg P \vee \neg Q$

$\neg(P \vee Q)$ is equivalent to $\neg P \wedge \neg Q$

Commutative Laws

$P \wedge Q$ is equivalent to $Q \wedge P$

$P \vee Q$ is equivalent to $Q \vee P$

Associative Laws

$P \wedge (Q \wedge R)$ is equivalent to $(P \wedge Q) \wedge R$

$P \vee (Q \vee R)$ is equivalent to $(P \vee Q) \vee R$

Idempotent Laws

$P \wedge P$ is equivalent to P

$P \vee P$ is equivalent to P

Distributive Laws

$P \wedge (Q \vee R)$ is equivalent to $(P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R)$ is equivalent to $(P \vee Q) \wedge (P \vee R)$

Absorption Laws

$P \vee (P \wedge Q)$ is equivalent to P

$P \wedge (P \vee Q)$ is equivalent to P

Double Negation Law

$\neg\neg P$ is equivalent to P

Tautology Laws

$P \wedge (\text{a tautology})$ is equivalent to P

$P \vee (\text{a tautology})$ is a tautology

$\neg(\text{a tautology})$ is a contradiction

Contradiction Laws

$P \wedge (\text{a contradiction})$ is a contradiction

$P \vee (\text{a contradiction})$ is equivalent to P

$\neg(\text{a contradiction})$ is a tautology

Conditional Laws

$P \rightarrow Q$ is equivalent to $\neg P \vee Q$

$P \rightarrow Q$ is equivalent to $\neg(P \wedge \neg Q)$

Contrapositive Laws

$P \rightarrow Q$ is equivalent to $\neg Q \rightarrow \neg P$

Quantifier Negation Laws

$\neg\exists x P(x)$ is equivalent to $\forall x \neg P(x)$

$\neg\forall x P(x)$ is equivalent to $\exists x \neg P(x)$