Math 300 B, C - Spring 2013
Midterm Exam
April 22, 2013

Name: ___________________________  Student ID no.: _____________

Signature: ___________________________  Section: _____________

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• Complete all five questions.
• You have 50 minutes to complete the exam.
1. For sets $A, B,$ and $C,$ show the following identities using logic symbols and equivalences.

(a) $A \setminus (A \cap B) = A \setminus B$

(b) $(A \setminus B) \setminus C = A \setminus (B \cup C)$

(c) $(A \setminus B) \cap (B \setminus A) = \emptyset$
2. Write useful contrapositives of each of the following sentences. All variables represent integers.

(a) If $xy = 3$ and $x < y$, then $x = 1$ and $y = 3$.

(b) If $x$ is even or $y$ is odd, then $x(y - 1)$ is even.

(c) If there exists a prime $p$ such that $p^2$ divides $x$, then $x$ is not squarefree. (Your contrapositive should incorporate a “for all” statement.)
3. Simplify the following expressions. Justify your results by showing a sequence of equivalent expressions connecting the original expression with your final one.

(a) \((P \lor Q) \lor \neg(\neg P \lor \neg R)\)

(b) \((P \lor \neg(\neg P \land \neg Q)) \land \neg((\neg P \land R) \lor (R \land \neg R))\)
4. Write a truth table for the statement $\neg P \land (Q \lor P)$

5. Find a formula using only $\neg$ and $\land$ that is equivalent to $(P \to Q) \to \neg(Q \lor P)$. 
DeMorgan’s laws
\[ \neg(P \land Q) \text{ is equivalent to } \neg P \lor \neg Q \]
\[ \neg(P \lor Q) \text{ is equivalent to } \neg P \land \neg Q \]

Commutative Laws
\[ P \land Q \text{ is equivalent to } Q \land P \]
\[ P \lor Q \text{ is equivalent to } Q \lor P \]

Associative Laws
\[ P \land (Q \land R) \text{ is equivalent to } (P \land Q) \land R \]
\[ P \lor (Q \lor R) \text{ is equivalent to } (P \lor Q) \lor R \]

Idempotent Laws
\[ P \land P \text{ is equivalent to } P \]
\[ P \lor P \text{ is equivalent to } P \]

Distributive Laws
\[ P \land (Q \lor R) \text{ is equivalent to } (P \land Q) \lor (P \land R) \]
\[ P \lor (Q \land R) \text{ is equivalent to } (P \lor Q) \land (P \lor R) \]

Absorption Laws
\[ P \lor (P \land Q) \text{ is equivalent to } P \]
\[ P \land (P \lor Q) \text{ is equivalent to } P \]

Double Negation Law
\[ \neg\neg P \text{ is equivalent to } P \]

Conditional Laws
\[ P \to Q \text{ is equivalent to } \neg P \lor Q \]
\[ P \to Q \text{ is equivalent to } \neg(P \land \neg Q) \]

Contrapositive Laws
\[ P \to Q \text{ is equivalent to } \neg Q \to \neg P \]

Quantifier Negation Laws
\[ \neg\exists x P(x) \text{ is equivalent to } \forall x \neg P(x) \]
\[ \neg\forall x P(x) \text{ is equivalent to } \exists x \neg P(x) \]

Sets
\[ A = B \iff ((x \in A) \iff (x \in B)) \]
\[ x \in A \cup B \iff ((x \in A) \lor (x \in B)) \]
\[ x \in A \cap B \iff ((x \in A) \land (x \in B)) \]
\[ x \in A \setminus B \iff (x \in A) \land (x \notin B) \]

Tautology Laws
\[ P \land (a \text{ tautology}) \text{ is equivalent to } P \]
\[ P \lor (a \text{ tautology}) \text{ is a tautology} \]
\[ \neg(a \text{ tautology}) \text{ is a contradiction} \]

Contradiction Laws
\[ P \land (a \text{ contradiction}) \text{ is a contradiction} \]
\[ P \lor (a \text{ contradiction}) \text{ is equivalent to } P \]
\[ \neg(a \text{ contradiction}) \text{ is a tautology} \]