Math 300 A - Spring 2012
Final Exam
June 4, 2012

Name: ___________________________       Student ID no.: ________________

Signature: ___________________________       Section: ____________

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- Complete all six questions.
- You have 110 minutes to complete the exam.
1. Assign “true” or “false” to each of the following statements. No justification need be given.

(a) If there is a function \( f : \mathbb{Z} \to A \), then \( A \) is countable.

(b) The function \( g : \mathbb{R} \to \mathbb{R} \) defined by \( g(x) = x^3 \) is a bijection.

(c) If \( A \) and \( B \) are sets, then \( B \) and \( A \setminus B \) are disjoint.

(d) If \( A \) is a set, and \( D \subseteq A \times A \), then \( D \) is a relation.

(e) If \( f : \mathbb{Z} \to \mathbb{Z} \), and \( f \) is onto, then \( f \) is one-to-one.

(f) There exist one-to-one functions from \( \mathbb{R} \) to \( \mathbb{Z} \).

(g) The set \( \{-5, 3, 4\} \) is an element of \( \mathcal{P}(\mathbb{Z}) \).

(h) Every subset of \( \mathbb{Q} \) is countable.
2. Let $A = \mathcal{P}(\mathbb{R})$. Define $f : \mathbb{R} \to A$ by the formula

$$f(x) = \{y \in \mathbb{R} : y^2 < x\}.$$ 

(a) Is $f$ one-to-one? Prove your answer.

(b) Is $f$ onto? Prove your answer.
3. Let $R$ be a relation on $\mathbb{Q}$ defined by $(p/q, r/s) \in R \Leftrightarrow ps = qr$. Show that $R$ is an equivalence relation.
4. Give a proof by induction that $6$ divides $n^3 - n$ for all $n \in \mathbb{Z}_{\geq 0}$. 
5. Suppose $f : A \to C$ and $g : B \to C$. Prove that if $A$ and $B$ are disjoint, then

$$f \cup g : A \cup B \to C.$$