Math 126 C - Spring 2008
Mid-Term Exam Number One
April 24, 2008

Name: ____________________________  Section: ____________

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- Complete all questions.

- You may use a scientific, non-graphing calculator during this examination. Other electronic devices are not allowed, and should be turned off for the duration of the exam.

- If you use a trial-and-error or guess-and-check method, or read a numerical solution from a graph on your calculator, when an algebraic method is available, you will not receive full credit.

- You may use one hand-written 8.5 by 11 inch page of notes.

- Show all work for full credit.

- You have 50 minutes to complete the exam.
1. Let \( f(x) = \frac{1}{3 - \frac{1}{4}x} \).

(a) Find the second-order Taylor polynomial \( T_2(x) \) for \( f(x) \) based at \( b = 0 \).

(b) Give a bound on the error \( |f(x) - T_2(x)| \) for \(-0.5 \leq x \leq 0.5\).
2. Let

\[ h(x) = \int_0^x t \cos t^3 \, dt \]

(a) Find the first four non-zero terms of the Taylor series based at \( b = 0 \) for \( h(x) \).

(b) Use your answer to part (a) to compute \( h^{(20)}(0) \).
3. Find the first three non-zero terms of the Taylor series for

\[ g(x) = \frac{x}{2 + x^2} \]
4. Consider the polar curve defined by the equation

\[ r = \theta (12 - \theta) \]

for \( 0 \leq \theta \leq 12 \). The curve is shown in the figure below.

(a) Find the slope of the tangent line to this curve at \( \theta = \frac{\pi}{2} \).

(b) Find the value of \( \theta \) corresponding to the self-intersection point indicated by the arrow.
5. There are infinitely many unit vectors which are orthogonal to the vector \((3, -2, 4)\). Give an example of one of them.