Math 125 H - Winter 2010
Mid-Term Exam Number Two
February 25, 2010
Answers

1. (a) Use a basic substitution, then use partial fractions.

$$\int \frac{\sqrt{x}}{x-9}\,dx = 2\sqrt{x} - 3\ln|\sqrt{x}+3| + 3\ln|\sqrt{x}-3| + C.$$

   (b) Write in terms of $\sin x$ and $\cos x$.

$$\int \frac{\tan^5 x}{\sec^8 x}\,dx = \frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C.$$

2. (a) Partial fractions.

$$\int \frac{x+3}{(x+1)(x+2)^2}\,dx = 2\ln|x+1| - 2\ln|x+2| + \frac{1}{x+2} + C.$$

   (b) Use a trig. substitution.

$$\int \frac{x^5}{\sqrt{1-x^2}}\,dx = -\frac{1}{5}(1-x^2)^{5/2} + \frac{2}{3}(1-x^2)^{3/2} - \sqrt{1-x^2} + C.$$

3. (a) Integration by parts.

$$\int x^2\ln(x^2+1)\,dx = \frac{1}{3}x^3\ln(x^2+1) - \frac{2}{9}x^3 + \frac{2}{3}x - \frac{2}{3}\tan^{-1}x + C.$$

   (b) Complete the square, then use a trig. substitution, or a basic substitution and the table of known integrals.

$$\int \frac{x}{\sqrt{2+2x-x^2}}\,dx = \sin^{-1}\frac{x-1}{\sqrt{3}} - \sqrt{3-(x-1)^2} + C.$$

4. Use a trig. substitution, then evaluate the limit.

$$\int_1^\infty \frac{dx}{x\sqrt{x^2+1}} = -\ln(\sqrt{2}-1).$$

5. (a) With δ representing density, and g the acceleration due to gravity, the work is equal to

$$\delta g\int_{-r}^{0}(-y)\pi(r^2-y^2)\,dy = \frac{1}{4}\pi\delta g r^4.$$

   (b) Doubling r will increase the work by a factor of $2^4 = 16$.