1. (a) \[ -\frac{1}{24} \cos^4 x + \frac{1}{26} \cos^6 x + C \]
(b) A \( u \)-substitution and integration by parts get you to
\[ \frac{1}{3} \sin^3 x \ln(\sin x) - \frac{1}{9} \sin^3 x + C \]

2. (a) Trig substitution \((x = 2 \sec \theta)\) to get
\[ 2^5 \left( \frac{1}{5} \frac{(x^2 - 4)^{5/2}}{2^5} + \frac{2}{3} \frac{(x^2 - 4)^{3/2}}{2^3} + \frac{\sqrt{x^2 - 4}}{2} \right) + C \]
(b) Long division, then factor the denominator and use partial fractions to get
\[ \frac{1}{2} x^2 + x + \frac{27}{5} \ln |x - 3| + \frac{8}{5} \ln |x + 2| + C \]

3. (a) Use \( u = x^4 \) and then integration by parts to get
\[ \frac{1}{4} x^4 e^{x^4} - \frac{1}{4} e^{x^4} + C \]
(b) Complete the square and use \( x + 3 = \sqrt{11} \tan \theta \) to get
\[ \frac{1}{\sqrt{11}} \tan^{-1} \left( \frac{x + 3}{\sqrt{11}} \right) + C \]

4. (a) Use the substitution \( u = e^x - 1 \) and then partial fractions. The antiderivative is
\[ \ln |e^x - 1| - \ln |e^x| + C \]
which is divergent as \( x \to 0 \) so the integral is divergent.
(b) Using \( x = \tan \theta \) gets the antiderivative
\[ -\frac{\sqrt{x^2 + 1}}{x} + C \]
The limit is
\[ -1 + \frac{\sqrt{5}}{2} \]

5. The integral is
\[ \int_0^{32} 900(9.8) \pi y^{2/5} (35 - y) \, dy \]
which evaluates to 41,378,545.1 J.

6. The average value of this function is
\[ \frac{1}{3} k + \frac{1}{2k} \]
This function of \( k \) is minimized when \( k = \sqrt{\frac{3}{2}} \).