Math 125 G - Winter 2009
Mid-Term Exam Number One
January 29, 2009
Answers with Suggestions

1. (a) $\frac{1}{2}(\ln x)^2 + C$ (Use the substitution $u = \ln x$.)

   (b) $\frac{9\pi}{2}$ (Factor 4 out of the quadratic expression, and use the fact that the resulting integral is the area of part of a circle).

   (c) $\frac{2}{25}(x^5+3)^{5/2} - \frac{2}{5}(x^5+3)^{3/2} + C$ (Try the substitution $u = x^5 + 3$.)

   (d) $\frac{1}{2}\ln(x^2+1) + 4\tan^{-1}(x) + C$ (Write the integrand as a sum of two fractions.)

2. $\frac{8\pi}{3}$ (The "washer" method works well.)

3. $\frac{2\pi}{45}$ (Using the shell method results in a single integral.)

4. By expressing each summand as a rectangle, and drawing appropriate figures, we can conclude that

$$\int_1^{n+1} \frac{dx}{x^2+1} < \sum_{i=1}^{n} \frac{1}{i^2+1} < \int_0^n \frac{dx}{x^2+1}$$

   and so, by evaluating the integrals, we have

$$\tan^{-1}(n+1) - \tan^{-1}(1) < \sum_{i=1}^{n} \frac{1}{i^2+1} < \tan^{-1}(n) - \tan^{-1}(0),$$

   which simplifies to

$$\tan^{-1}(n+1) - \frac{\pi}{4} < \sum_{i=1}^{n} \frac{1}{i^2+1} < \tan^{-1}(n).$$

5. $m = 6$. (Begin by finding the intersection of the two lines with $y = x^2$. Conclude that the area is given by the sum

$$A = \int_0^m mx\,dx + \int_m^{2m} (2mx - x^2)\,dx.$$

   Evaluate these integrals, sum, set the result equal to 252 and solve for m.)

6. Using the "washer" method, we have that the volume, V, is given by

$$V = \int_a^b \left(\pi(f(x)+k)^2 - \pi(g(x)+k)^2\right)\,dx$$

$$= \pi \int_a^b (f(x) - g(x))(f(x) + g(x) + 2k)\, dx$$

$$= \pi \int_a^b (f(x) - g(x))(f(x) + g(x))\, dx + k \left(2\pi \int_a^b (f(x) - g(x))\, dx \right).$$

which is of the form A+kB, with A and B constants.