1. Evaluate each of the following indefinite integrals.

(a) \[ \int \frac{x^2}{x^3 + 5} \, dx = \frac{1}{3} \ln |x^3 + 5| + C \]

(b) \[ \int (x^2 + 3)^2 \, dx = \int (x^4 + 6x^2 + 9) \, dx = \frac{1}{5}x^5 + 2x^3 + 9x + C \]

(c) \[ \int x^3 \sqrt{x^2 + 4} \, dx = \frac{1}{5}(x^2 + 4)^{5/2} - \frac{4}{3}(x^2 + 4)^{3/2} + C \]

(d) \[ \int \frac{1}{x \ln x} \, dx = \ln |\ln x| + C \]

2. Alice falls from a plane at an altitude of 3000 meters. She falls in such a way that she is accelerating at a rate of \[-9.8 + 0.3t\] m/s\(^2\) t seconds after the start of her fall. Assume her initial velocity is zero.

(a) What is her velocity after 6 seconds?

Her velocity will be the integral of her rate of acceleration from \(t = 0\) to \(t = 6\):
\[ v = \int_0^6 (-9.8 + 0.3t) \, dt = (-9.8t + 0.15t^2)|_0^6 = -9.8(6) + 0.15(6^2) = -53.4 \text{ m/s}. \]

(b) How far off the ground will she be after falling for 6 seconds?

Her velocity after \(t\) seconds is given by
\[ v = -9.8t + 0.15t^2 \]

Integrating this, we find her position is
\[ h = -4.9t^2 + 0.05t^3 + C \]

Since \(h = 3000\) when \(t = 0\), \(C = 3000\), so her height after \(t\) seconds is
\[ h = -4.9t^2 + 0.05t^3 + 3000 \]

and so after 6 seconds, she’ll be
\[ h = -4.9(6^2) + 0.05(6^3) + 3000 = 2834.4 \text{ meters off the ground}. \]

3. The graph of \(f(x)\) is given below. Let \(A(x) = \int_0^x f(t) \, dt\).

\[ \text{Evaluate each of the following:} \]
(a) $A(2) = (2)(2) = 4$
(b) $A'(3) = 1$
(c) $A(6) = (2)(2) + 2 - \frac{1}{2} - 1 = \frac{9}{2}$
(d) $A(4) - A(3) = \frac{1}{2}$

4. Let $R$ be the region in the first quadrant bounded by $y = 2 - x^2$, $y = x^2$, and the $y$-axis.
(a) Find the volume of the solid of revolution created by revolving $R$ about the $y$-axis.
The curves $y = x^2$ and $y = 2 - x^2$ intersect in the first quadrant at the point $(1, 1)$.
$$V = \int_0^1 2\pi x(2 - x^2 - x^2) \, dx = \int_0^1 2\pi(2x - 2x^3) \, dx = 2\pi (x^2 - \frac{1}{2} x^4) \bigg|_0^1 = 2\pi (1 - \frac{1}{2}) = \pi.$$  

(b) Find the volume of the solid of revolution created by revolving $R$ about the $x$-axis.
$$V = \int_0^1 (\pi(2 - x^2)^2 - \pi(x^2)^2) \, dx = \pi \int_0^1 (4 - 4x^2 + x^4 - x^4) \, dx$$
$$= \pi \int_0^1 (4 - 4x^2) \, dx = \pi \left( 4x - \frac{4}{3}x^3 \right) \bigg|_0^1 = \pi \left( 4 - \frac{4}{3} \right) = \frac{8}{3}\pi$$

5. Let $R$ be the region bounded by $y = x$, $y = \ln(x^2 + 1)$, and $x = 3$. The curves are shown in the figure.

Set up an integral that gives the volume of the solid of revolution created by revolving $R$ about the line $x = 5$. DO NOT EVALUATE THE INTEGRAL.
The method of cylindrical shells is the easiest way to go on this problem:
$$V = \int_0^3 2\pi(5 - x)(x - \ln(x^2 + 1)) \, dx$$

6. Here is a graph of $y = e^{\cos x}$ on the interval $0 \leq x \leq 3$:  

![Graph of $y = e^{\cos x}$ on the interval $0 \leq x \leq 3$]
Use the midpoint rule with \( n = 3 \) to approximate the value of the following integral:

\[
\int_{0}^{3} e^\cos{x} \, dx
\]

\[
\int_{0}^{3} e^\cos{x} \, dx \approx (1)e^{\cos(0.5)} + (1)e^{\cos(1.5)} + (1)e^{\cos(2.5)} = 3.927193071...
\]

Note that you must have your calculator in radian mode in order to correctly calculate \( \cos(0.5) = 0.87758256... \) etc.

7. Find the value of \( m \) so that the region bounded by \( y = \sqrt{x} \) and \( y = mx \) has an area of 4.

The curves \( y = \sqrt{x} \) and \( y = mx \) intersect at \( x = \frac{1}{m^2} \). The area of the region bounded by these curves is

\[
\int_{0}^{1/m^2} (\sqrt{x} - mx) \, dx = \left( \frac{2}{3}x^{3/2} - \frac{1}{2}mx^2 \right)_{0}^{1/m^2} = \frac{1}{6m^3}
\]

So, if the area equals 4, then we have

\[
4 = \frac{1}{6m^3}
\]

from which we find

\[
m = \frac{1}{\sqrt[3]{24}} = \frac{1}{2\sqrt[3]{3}} = 0.3466806372...
\]