How do I get sinusoidal functions into the "standard" form?

When confronted with a sinusoidal function, it is helpful to have it (or get it into) the form

$$f(x) = A\sin\left(\frac{2\pi}{B}(x-C)\right) + D$$

since we are able to read off the parameters A, B, C, and D; these parameters determine the shape and location of the graph of function. We are always able to put sinusoidal functions into this form with $A > 0$, and $B > 0$.

Frequently, sinusoidal functions do not appear to us in this form, but we can rewrite in that form. The following are three examples of how this goes.

1. $f(x) = \sin(3x+1)$

   We factor out the 3 from the expression $3x+1$ to get

   $$f(x) = \sin\left(3\left(x+\frac{1}{3}\right)\right).$$

   We want $\frac{2\pi}{B} = 3$, so $B = \frac{2}{3}\pi$. Thus,

   $$f(x) = \sin\left(\frac{2\pi}{\frac{2}{3}\pi}\left(x+\frac{1}{3}\right)\right).$$

   And we can see that $A = 1, B = \frac{2}{3}\pi, C = -\frac{1}{3}, D = 0$.

2. $f(x) = \sin(2-7x)$

   We can factor the -7:

   $$f(x) = \sin\left(-7\left(-\frac{2}{7}+x\right)\right).$$

   In order to end up with $B > 0$ we have to do something about that -7. We can use the following identity:

   $$\sin(-x) = \sin(x+\pi) \text{ for all } x.$$

   This gives us

   $$f(x) = \sin\left(-7\left(-\frac{2}{7}+x\right)\right) = \sin\left(7\left(-\frac{2}{7}+x\right)+\pi\right) = \sin\left(7\left(-\frac{2}{7}+x+\frac{\pi}{7}\right)\right)$$
   $$= \sin\left(7\left(x-\left(\frac{2}{7}-\frac{\pi}{7}\right)\right)\right) = \sin\left(\frac{2\pi}{\frac{2}{7}\pi}\left(x-\left(\frac{2}{7}-\frac{\pi}{7}\right)\right)\right).$$

   So $A = 1$,$B = \frac{2}{7}\pi$, $C = \frac{2-\pi}{7}$, $D = 0$.

3. $f(x) = -\frac{3}{5}\sin(5-x)$

   We can use the identity

   $$-\sin(x) = \sin(-x) \text{ for all } x$$

   to get a positive A value:

   $$f(x) = \frac{3}{5}\sin(-(5-x)) = \frac{3}{5}\sin(x-5) = \frac{3}{5}\sin\left(\frac{2\pi}{2\pi}(x-5)\right).$$

   So $A = \frac{3}{5}, B = 2\pi, C = 5, D = 0$.