Comment on OEIS A164624

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This sequence is a subset of primes, numbers p such that p and $p + \lfloor \frac{p}{2} \rfloor + \lfloor \frac{p}{3} \rfloor$ are both primes. Looking at the sequence modulo 9, we see that the congruence classes of 7 and 8 are completely missing. For the sequence modulo 12, we also see that congruence classes of 5 and 7 are missing as well. We have the following theorem on the congruence classes of A164624.

Theorem. If $n \in$A164624, then $n \not\equiv 0, 3, 6, 7, 8 \pmod 9$, and $n \not\equiv 0, 2, 3, 4, 5, 6, 7, 8, 9, 10 \pmod{12}$ except that $2, 3 \in$A164624.

Proof. Let $p \in$A164624. We prove by contrapositive for the following cases:

- $p \not\equiv 5, 7 \pmod{12}$
- $p \not\equiv 7, 8 \pmod 9$

Suppose that $p \equiv 5 \pmod{12}$ is prime. Then we can write $p = 12k + 5 > 0$, some $k \geq 0$, then

$$\begin{aligned} p + \left\lfloor \frac{p}{2} \right\rfloor + \left\lfloor \frac{p}{3} \right\rfloor &= 12k + 5 + \left\lfloor \frac{12k+5}{2} \right\rfloor + \left\lfloor \frac{12k+5}{3} \right\rfloor \\ &= 12k + 5 + 6k + 2 + 4k + 1 \\ &= 22k + 8 \end{aligned}$$

$22k + 8$ is even, so it is not prime and and therefore no in A162624.

Similarly if $p \equiv 7 \pmod{12}$, write $p = 12k + 7$, some $k \geq 0$, then

$$\begin{aligned} p + \left\lfloor \frac{p}{2} \right\rfloor + \left\lfloor \frac{p}{3} \right\rfloor &= 12k + 7 + \left\lfloor \frac{12k+7}{2} \right\rfloor + \left\lfloor \frac{12k+7}{3} \right\rfloor \\ &= 12k + 7 + 6k + 3 + 4k + 2 \\ &= 22k + 12 \end{aligned}$$

$22k + 12$ is not prime and not in the sequence for the same reason.

Suppose that $p \equiv 7 \pmod 9$ is prime; write $p = 9k + 7$, and k is even since otherwise p is even. Write $k = 2m,\ m \geq 0$. Similarly

$$\begin{aligned} p + \left\lfloor \frac{p}{2} \right\rfloor + \left\lfloor \frac{p}{3} \right\rfloor &= 9(2m) + 7 + \left\lfloor \frac{9(2m)+7}{2} \right\rfloor + \left\lfloor \frac{9(2m)+7}{3} \right\rfloor \\ &= 18m + 7 + 9m + 3 + 6m + 2 \\ &= 33m + 12 \equiv 0 \pmod 3 \end{aligned}$$

Hence $p + \lfloor \frac{p}{2} \rfloor + \lfloor \frac{p}{3} \rfloor$ is not prime.

Suppose now that $p \equiv 8 \pmod 9$ is prime, we can write $p = 9k + 8$, for some positive odd number k, so $k = 2m + 1$. Rewrite the sequence formula:

$$\begin{aligned} p + \left\lfloor \frac{p}{2} \right\rfloor + \left\lfloor \frac{p}{3} \right\rfloor &= 9(2m+1) + 8 + \left\lfloor \frac{9(2m+1)+8}{2} \right\rfloor + \left\lfloor \frac{9(2m+1)}{3} \right\rfloor \\ &= 18m + 9 + 8 + 9m + 4 + 4 + 6m + 3 \\ &= 33m + 27 \equiv 0 \pmod 3 \end{aligned}$$

Hence if $p \equiv 7, 8 \pmod 9$ or $p \equiv 5, 7 \pmod{12}$, then $p + \lfloor \frac{p}{2} \rfloor + \lfloor \frac{p}{3} \rfloor$ is not prime, and so the theorem, being the contrapositive, is true as well.

Now we consider the remaining cases. If a number $n \equiv 0, 3, 6 \pmod 9$ and $n \neq 3$. Clearly it is not prime since we can represent $n = 9k + 0$, or $n + 9k + 3$, or $n = 9k + 6$, and they are divisible by 3, so they are not in the sequence. If a number $n \equiv 0, 2, 3, 4, 6, 8, 9, 10 \pmod{12}$. and $n \neq 2, 3$. Then n is divisible by 2 if $n \equiv 2, 4, 6, 8, 10 \pmod{12}$, and n is divisible by 3 if $n \equiv 3, 6, 9 \pmod{12}$. Hence the theorem is proved. $\square$

We see that there are two pronouced spectral lines at 3671Hz and 4897Hz, which corresponds to $\frac{44100}{12}$, $\frac{44100}{9}$, respectively.