The following gives a good estimate of the difference sequence of A050504.

The result shows that the differences of the sequence change by at most one from term to term (in other words, the second differences are all $0, 1$ or -1).

For $x \in \mathbb{R}$, define $\lfloor x \rfloor$ to be the greatest integer less than or equal to x. Define $\{x\} = x - \lfloor x \rfloor$, the fractional part of x (at least when x is positive).

Lemma 1: For $a, x \in \mathbb{R}$, $x, a > 0$, we have

$$\lfloor a \rfloor \le \lfloor x + a \rfloor - \lfloor x \rfloor \le 1 + \lfloor a \rfloor.$$

Proof: Let $a, x \in \mathbb{R}$, $x, a > 0$.
Let $A = \lfloor x + a \rfloor - \lfloor x \rfloor$.
Then $A = x + a - \{x + a\} - x + \{a\} = a + \{x\} - \{x + a\}$.
So $A \le a + x < a + 1$.
Since $A \in \mathbb{Z}$, $A \le \lfloor a + 1 \rfloor = \lfloor a \rfloor + 1$.
Also, $A > a - \{x + a\} > a - 1$ and so $A \ge 1 + \lfloor a - 1 \rfloor = \lfloor a \rfloor$. $\square$

Notice that, since $\lfloor 0.2 + 0.6 \rfloor - \lfloor 0.2 \rfloor = 0 = \lfloor 0.6 \rfloor$ and $\lfloor 0.2 + 0.9 \rfloor - \lfloor 0.2 \rfloor = 1 = \lfloor 0.9 \rfloor + 1$, the inequalities are best possible.

Lemma 2: For all real $n \ge 1$,

$$\frac{1}{n} - \frac{1}{2n^2} < \log(n+1) - \log n < \frac{1}{n}.$$

Proof: Let $f(x) = \log x$. Let $n \ge 1$. The first Taylor polynomial $P_1(x)$ for f based at n is

$$P_1(x) = f(n) + f'(n)(x - n) = \log n + \frac{1}{n}(x - n)$$

and, by Taylor's theorem,

$$\log x = P_1(x) + \frac{f''(\gamma)}{2!}(x - n)^2 = P_1(x) - \frac{1}{2\gamma^2}(x - n)^2$$

for some γ between n and x. Taking $x = n + 1$, we have

$$\begin{aligned} \log(n+1) &= \log n + \frac{1}{n} - \frac{1}{2\gamma^2} \\ &< \log n + \frac{1}{n} \end{aligned}$$

so $\log(n+1) - \log(n) < \frac{1}{n}$.
The second Taylor polynomial $P_2(x)$ for f based at n is

$$P_2(x) = f(n) + f'(n)(x - n) + \frac{f''(n)}{2!}(x - n)^2 = \log n + \frac{1}{n}(x - n) - \frac{1}{2n^2}(x - n)^2.$$

By Taylor's theorem,

$$\log x = P_2(x) + \frac{f'''(\gamma)}{3!}(x-n)^3 = P_2(x) + \frac{2}{3!\gamma^3}(x-n)^2$$

for some γ between n and x. Taking $x = n+1$, we have

$$\log(n+1) = \log n + \frac{1}{n} - \frac{1}{2n^2} + \frac{2}{3!\gamma^3}$$

so

$$\log(n+1) - \log n > \frac{1}{n} - \frac{1}{2n^2}. \quad \square$$

Theorem: Let n be a positive integer, and define $f(n) = \lfloor n \log n \rfloor$. Then

$$f(n+1) - f(n) = a + \lfloor \log(n+1) \rfloor, \text{ where } a \in \{1, 2\}.$$

Proof: Let n be a positive integer.
Let $A = \lfloor (n+1)\log(n+1) \rfloor - \lfloor n \log n \rfloor$.
Let $x = n \log n$ and $a = (n+1)\log(n+1) - n\log n = n(\log(n+1) - \log n) + \log(n+1)$.
Then $A = \lfloor x + a \rfloor - \lfloor x \rfloor$.
By the previous lemma,

$$1 - \frac{1}{2n} < n(\log(n+1) - \log n) < 1$$

so

$$1 - \frac{1}{2n} + \log(n+1) < a < 1 + \log(n+1).$$

Then, by Lemma 1,

$$\lfloor a \rfloor \le A \le 1 + \lfloor a \rfloor$$

and we have

$$1 + \lfloor a \rfloor \le 1 + \lfloor 1 + \log(n+1) \rfloor = 2 + \lfloor \log(n+1) \rfloor.$$

Also,

$$a > 1 + \log(n+1) - \frac{1}{2n}$$

so

$$\lfloor a \rfloor \ge 1 + \lfloor 1 + \log(n+1) - \frac{1}{2n} \rfloor = 2 + \lfloor \log(n+1) - \frac{1}{2n} \rfloor \ge 1 + \lfloor \log(n+1) \rfloor.$$

Hence,

$$1 + \lfloor \log(n+1) \rfloor \le A \le 2 + \lfloor \log(n+1) \rfloor,$$

that is,

$$1 + \lfloor \log(n+1) \rfloor \le \lfloor (n+1)\log(n+1) \rfloor - \lfloor n \log n \rfloor \le 2 + \lfloor \log(n+1) \rfloor. \quad \square$$