Writing Assignment 4 - Math 381 A - Dr. Matthew Conroy

Consider the following game played with a single, 6-sided die. All sides are equally likely, but the 3 has been replaced by a 4 (so the die has two 4s, no 3, and one each of $1, 2, 5$ and 6.)

> A player starts with a score of zero and throws the die repeatedly, adding the face that appears on each throw of the die to their score.
>
> If the score is ever a prime number less than 18 (i.e., $2, 3, 5, 7, 11, 13$ or 17), then the player loses.
>
> If the score reaches a value of 18 or greater, the player wins.

Model this game using a Markov chain.

Define a transition matrix with two absorbing states (one for winning, one for losing).

Use the transition matrix to determine the exact probability of the player winning the game (the exact probability will be a rational number).

Write a piece of code to generate the transition matrix: do not create the matrix by hand!

Be sure to clearly describe your process and thoroughly support all of your claims. Include all code in your writeup.

Note: Your calculations should be exact (i.e., the probability should be expressed as an integer fraction, not as a decimal approximation). I recommend using a system (like Sage, PARI/GP, Maple, etc.) that allows you to perform calculations with rational numbers exactly. Alternatively, you could figure out a way to perform the calculations entirely with integer operations, and then you can perform the calculations exactly on more systems. Matlab, in particular, does not by default handle rational numbers exactly, so if you use Matlab you will need to be very careful how you proceed. Do not use the **rat** function in Matlab to convert a floating-point result to an "exact" rational expression: your result will **not** be exact!