Writing Assignment 3 - Math 381 A - Winter 2019

There are two problems below.

You need to do exactly one of them.

Look at your student id number.

If the right-most digit of your student id number is odd, do problem #1.

If the right-most digit of you student id number is even, do problem #2.

1. Define a graph $G = (V, E)$ as follows.
   Let $V = \{1, 2, 3, \ldots, 12\}$.
   Define $E = \{(i, j) : i, j \in V, i \neq j, ij + 5$ is prime $\}$.
   Create and solve (using lpsolve) an IP to find the chromatic number of G, $\chi(G)$.
2. Define a graph $G = (V, E)$ as follows.
   Let $V = \{1, 2, 3, \ldots, 12\}$.
   Define $E = \{(i, j) : i, j \in V, i \neq j, i^2 + j^2 + 18$ is prime $\}$.
   Create and solve (using lpsolve) an IP to find the chromatic number of G, $\chi(G)$.

Be sure to give a complete explanation of your method of solution.

Explicitly list your objective function and all constraints in your IP.

Include *all* code you write to solve the problem, and *all* software output.

You are welcome to use any programming language(s).

Please include the following components in this format:

1. Problem statement
2. Description of solution method including the mathematical formulation of the IP you will be using. Explain your method thoroughly.
3. Code to generate the lpsolve input file
4. The lpsolve input file. Be sure to truncate it: give one or two examples of each type of constraint, then remove the others, and indicate the number of constraints of each type removed.
5. Information about how you ran lpsolve on the above file, including run time and machine used, and the lpsolve solution output. Be sure to truncate it: leave out all variables which are equal to zero, and indicate that you have done this (e.g., "All other variables equal zero.").
6. Answer the question.
7. Include a figure that illustrates the feasibility of coloring your graph with the minimum number of colors you are claiming.

Note: Suppose a and b are positive integers.

We say that a is a *divisor* of b if $b = ak$ for some integer k.

A *prime* is an integer greater than 1 that has no divisors other than 1 and itself.

The sequence of primes begins $2, 3, 5, 7, 11, \ldots$.

A note on presenting large lpsolve input files

In this problem, your lpsolve input file will be quite long (well over 100 lines). Rather than print out an include the whole thing, please just give one sample constraint of each type, with a note that there are a certain number of constraints of that type omitted. At the same time, please comment to say what the purpose of those constraints are. For example, your lpsolve input file might be presented like this:

```
min: +c_1+c_2+c_3+c_4+c_5+c_6+c_7+c_8+c_9+c_10+c_11+c_12;
(11 lines of the following type:
ensure that lower numbered colors are used before higher numbered colors)
c_2<=c_1;
.
.
c_12<=c_11;
(12 lines of the following type:
ensure that every vertex is colored with exactly one color)
+x_1_1+x_1_2+x_1_3+x_1_4+x_1_5+x_1_6+x_1_7+x_1_8+x_1_9+x_1_10+x_1_11+x_1_12=1;
.
.
(142 lines of the following type:
ensure that the c variables keep track of colors that are used)
x_1_1<= c_1;
.
.
(251 lines of the following type:
ensure vertices connected by edges are different colors)
x_1_1+x_2_1 <=1;
.
.
(ensure all variables are binary)
bin x_1_1,...,x_12_12,c_1,...,c_12;
```