MATH 300 A
Exam I
Winter 2017

Name ____

Student ID #___________________

HONOR STATEMENT

"I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam."

SIGNATURE:______________________________________

1	6	
2	4	
3	12	
4	10	
5	8	
Total	40	

- Your exam should consist of this cover sheet, followed by 5 problems on 4 pages. Check that you have a complete exam.
- You are allowed to use the sheet of axioms and elementary properties of integers on the last page of this exam. You are not allowed to use any other sources.
- In your proofs, you may use any item on the sheet of axioms and elementary properties and any definition introduced in lecture. You may also use any facts you know about specific integers (e.g., that $5 \cdot 7 = 35$ or that 3 divides 18 or that the only positive divisors of 10 are 1, 2, 5, and 10). All other claims should be justified.
- Turn your cell phone OFF and put it AWAY for the duration of the exam.

GOOD LUCK!

1. (6 points) Give a meaningful negation of each statement. You may use symbols like $\forall$ and $\exists$. $\boxed{\text{Please put a box around your final answer.}}$

   (a) There exists an integer m such that, for every integer n, $m|n$ or $m > n$.

   (b) For every $p > 0$, there exists $d > 0$ such that, for every real number x, if $|x - 3| < d$, then $|x^2 - 9| < p$.

2. (4 points) Suppose n and m are integers. Prove that, if $n + m > 10$, then $n > 5$ or $m > 5$.

3. (12 points) Suppose a, b, and c are integers and let R be the statement

$$R\text{: } c|a \text{ or } c|b \text{ only if } c|ab.$$

   (a) Rewrite R in "*if-then*" form and give its converse.
   R in "*if-then*" form:

   converse of R:

   (b) Is R true? If so, prove it. If not, give a counter-example that demonstrates that R is false.

   (c) Is the converse of R true? If so, prove it. If not, give a counter-example that demonstrates that the converse of R is false.

4. (10 points) Suppose a and b are integers. Prove that $a^2 + 3b + 4$ is even if and only if a and b have the same parity.

   If it is relevant, you may use without proof the fact that, for every pair of integers x and y,
$$(x + y)^2 = x^2 + 2xy + y^2.$$

5. (8 points) Prove that, for all $n \in \mathbb{Z}$, there exists a unique $m \in \mathbb{Z}$ such that $5n - 2m = 3m + 10$.

Axioms of the Integers (AIs)

Suppose a, b, and c are integers.

- **Closure:**

  $a + b$ and ab are integers.

- **Substitution of Equals:**

  If $a = b$, then $a+c = b+c$ and $ac = bc$.

- **Commutativity:**

  $a + b = b + a$ and $ab = ba$.

- **Associativity:**

  $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

- **The Distributive Law:**

  $a(b + c) = ab + ac$

- **Identities:**

  $a + 0 = 0 + a = a$ and $a \cdot 1 = 1 \cdot a = a$

  0 is called the *additive identity*

  1 is called the *multiplicative identity.*

- **Additive Inverses:**

  There exists a real number $-a$ such that $a + (-a) = (-a) + a = 0$.

- **Trichotomy:**

  Exactly one of the following is true: $a > 0$, $-a > 0$, or $a = 0$.

Elementary Properties of the Integers (EPIs)

Suppose a, b, c, and d are integers.

1. $a \cdot 0 = 0$
2. If $a + c = b + c$, then $a = b$.
3. $-a = (-1) \cdot a$
4. $(-a) \cdot b = -(a \cdot b)$
5. $(-a) \cdot (-b) = a \cdot b$
6. If $a \cdot b = 0$, then $a = 0$ or $b = 0$.
7. If $a \leq b$ and $b \leq a$, then $a = b$.
8. If $a < b$ and $b < c$, then $a < c$.
9. If $a < b$, then $a + c < b + c$.
10. If $a < b$ and $0 < c$, then $ac < bc$.
11. If $a < b$ and $c < 0$, then $bc < ac$.
12. If $a < b$ and $c < d$, then $a + c < b + d$.
13. If $0 \leq a < b$ and $0 \leq c < d$, then $ac < bd$.
14. If $a < b$, then $-b < -a$.
15. $0 \leq a^2$
16. If $ab = 1$, then either $a = b = 1$ or $a = b = -1$.

NOTE: Properties 8-14 hold if each $<$ is replaced with $\leq$.