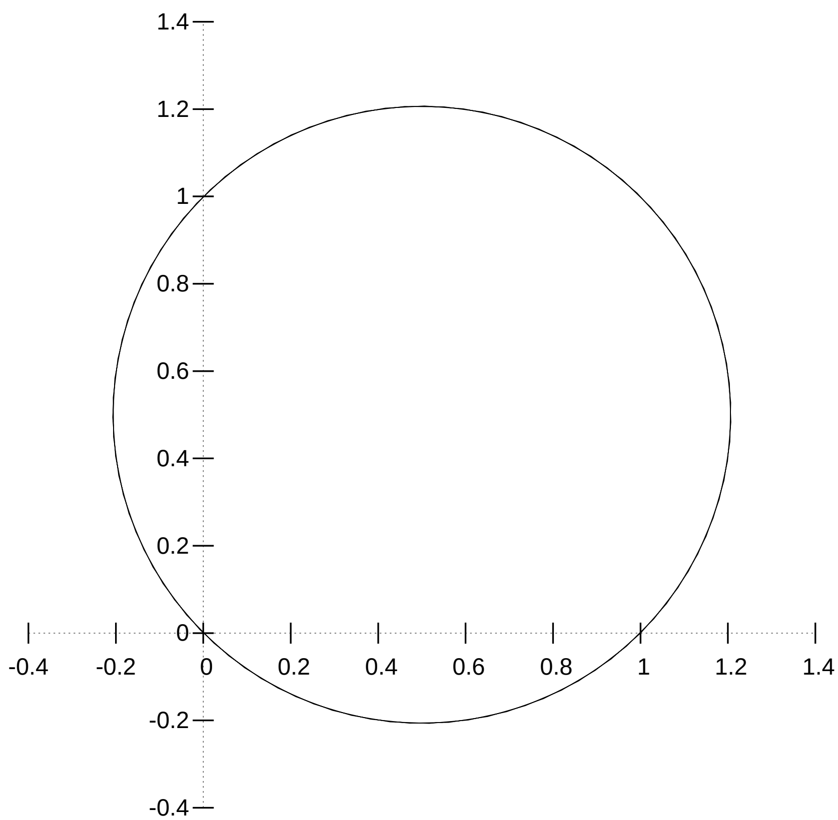
Polar curve examples

1. $r = \cos(\theta) + \sin(\theta)$

   This curve has Cartesian equation

   $$\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{1}{2}$$

   The curve is a circle.

   In general, the family

   $$r = A\cos(\theta) + B\sin(\theta)$$

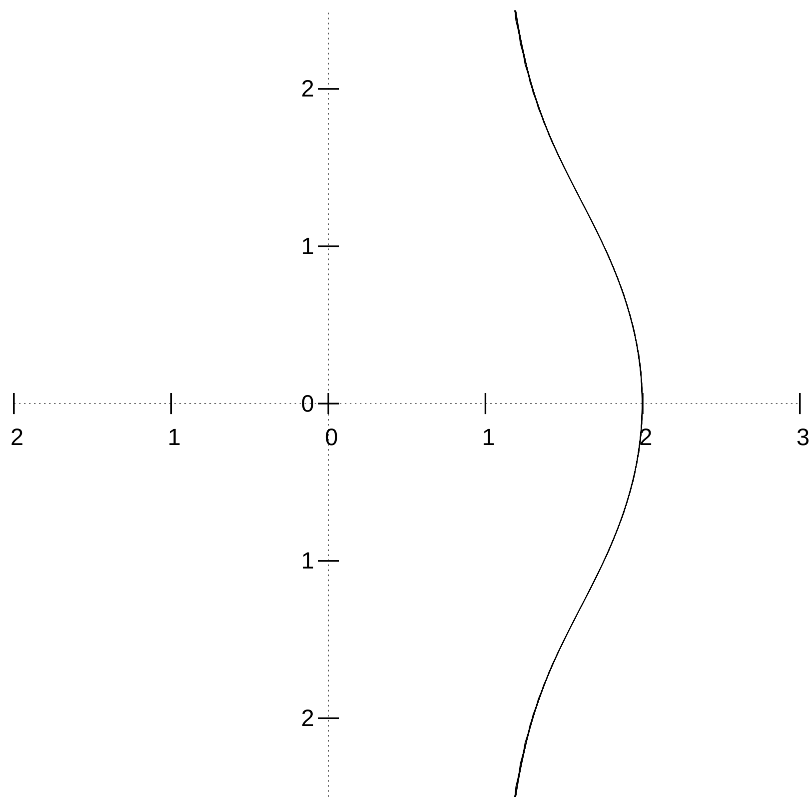
   consists of all circles through the origin.

2. $r = \cos(\theta) + \sec(\theta)$

This curve has Cartesian equation

$$x^3 + xy^2 - 2x^2 - y^2 = 0$$

and asymptote $x = 1$.

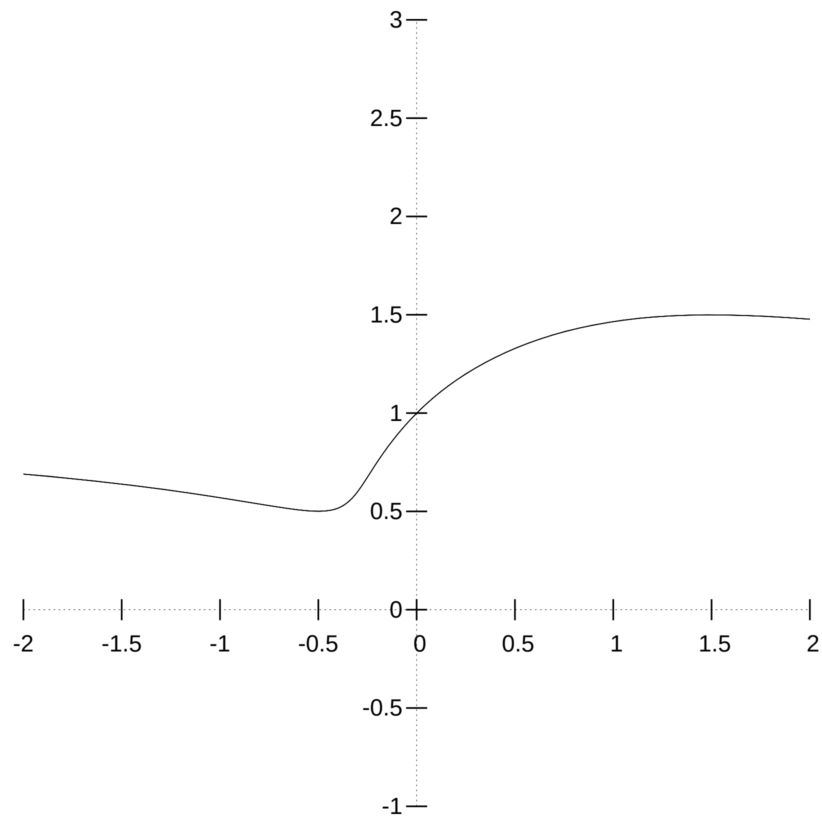


3. $r = \cos(\theta) + \csc(\theta)$

This curve has Cartesian equation

$$(y-1)(x^2+y^2) - xy = 0.$$

and asymptote $y = 1$.

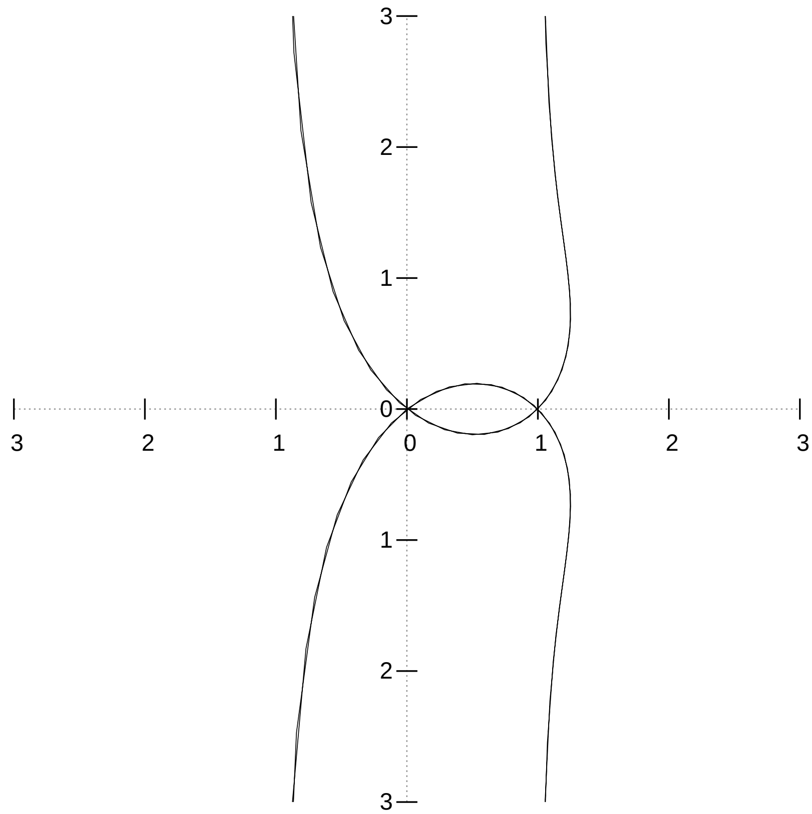


4. $r = \cos(\theta) + \tan(\theta)$

This curve has Cartesian equation

$$x^2(x^2 + y^2) - (x^2 + y^2)y^2 = 0$$

and asymptotes $x = \pm 1$.

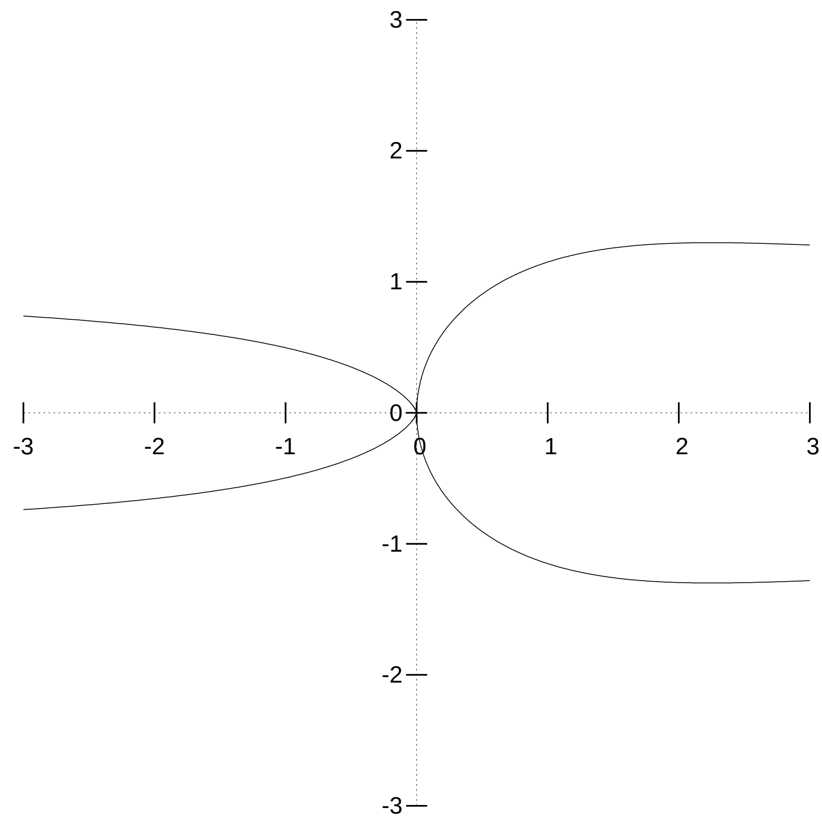


5. $r = \cos(\theta) + \cot(\theta)$

This curve has Cartesian equation

$$((x^2 + y^2)y - xy)^2 - x^2(x^2 + y^2) = 0$$

and asymptotes $y = \pm 1$.

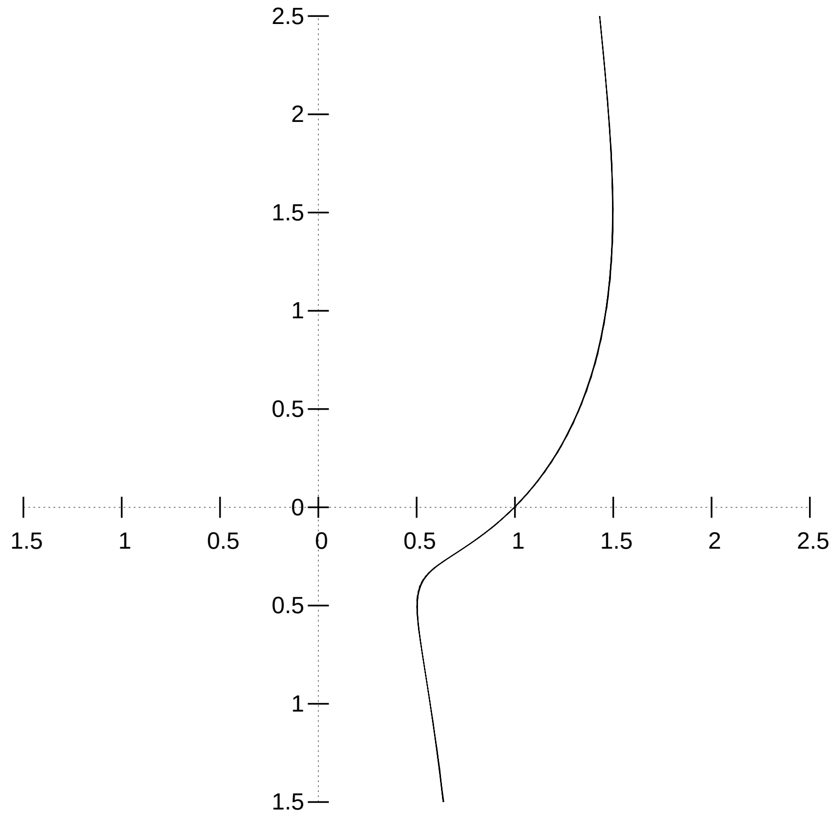


6. $r = \sin(\theta) + \sec(\theta)$

This curve has Cartesian equation

$$(x-1)(x^2+y^2) - xy = 0$$

and asymptote $x = 1$.

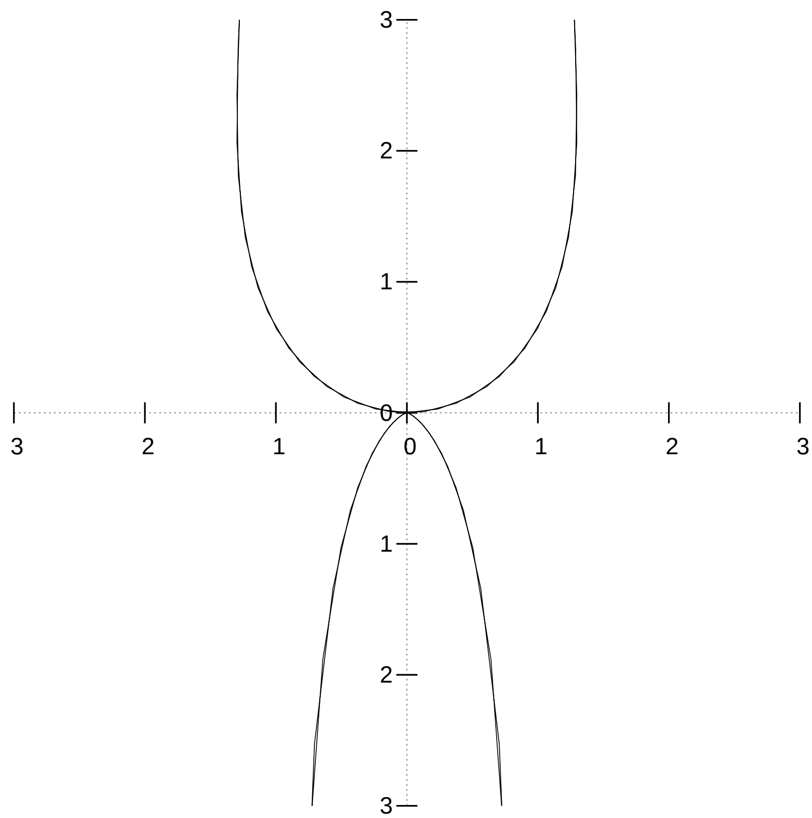


7. $r = \sin(\theta) + \tan(\theta)$

This curve has Cartesian equation

$$(x^2 + y^2)(x^4 + x^2y^2 - 2x^2y) - y^4 = 0$$

and asymptotes $x = \pm 1$.

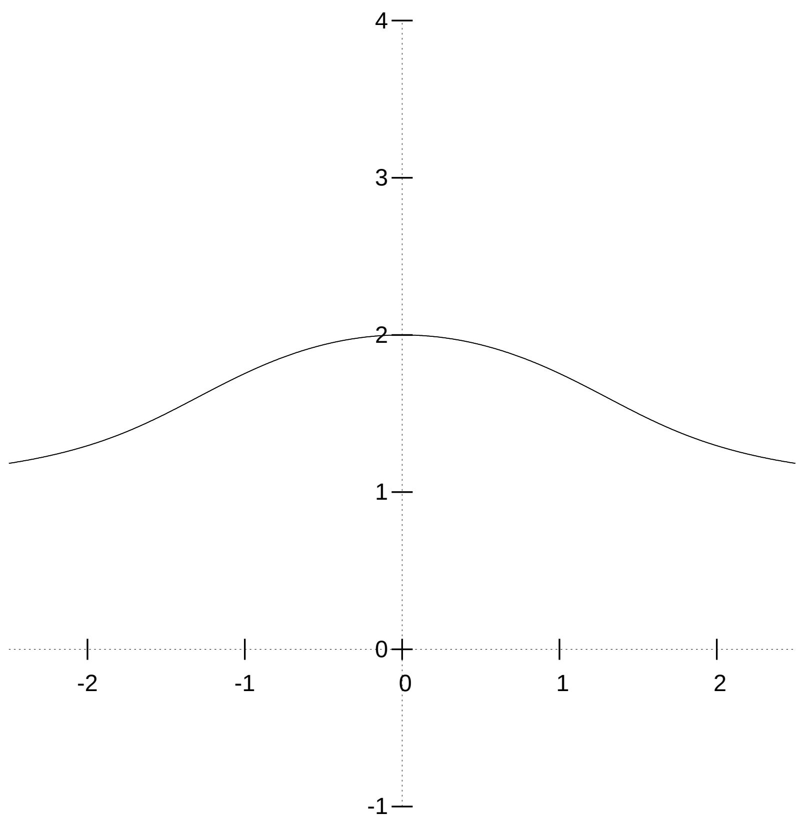


8. $r = \sin(\theta) + \csc(\theta)$

This curve has Cartesian equation

$$(x^2 + y^2)(y - 1) - y^2 = 0$$

and asymptote $y = 1$.

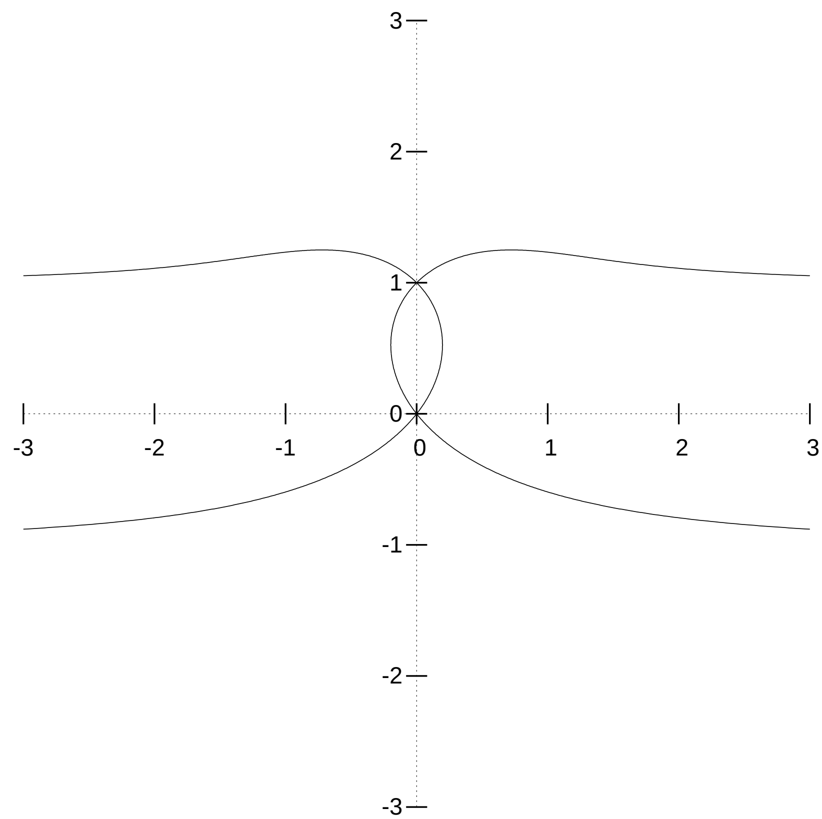


9. $r = \sin(\theta) + \cot(\theta)$

This curve has Cartesian equation

$$((x^2 + y^2)y - y^2)^2 - (x^2 + y^2)x^2 = 0$$

and asymptotes $y = \pm 1$.

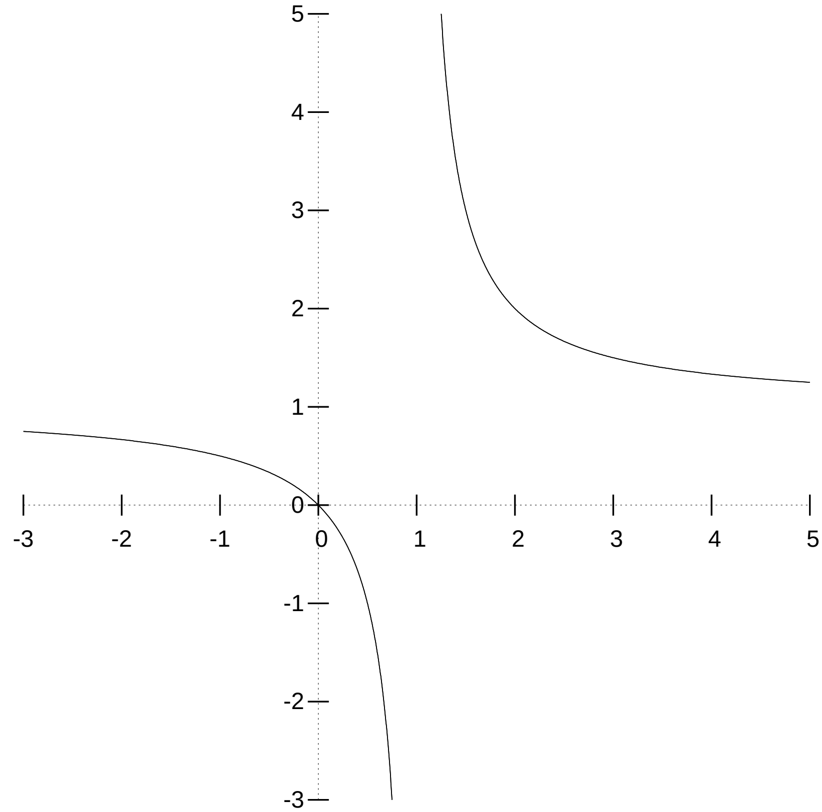


10. $r = \sec(\theta) + \csc(\theta)$

Thus curve has Cartesian equation

$$y = \frac{x}{x-1}$$

and asymptotes $x = 1$ and $y = 1$.

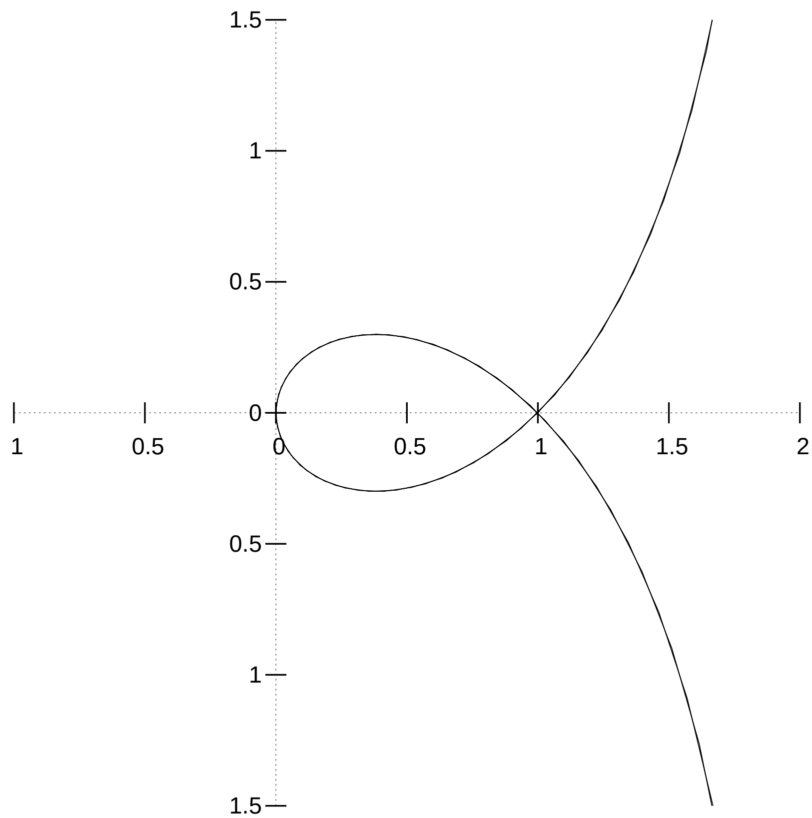


11. $r = \sec(\theta) + \tan(\theta)$

This curve has Cartesian equation

$$x^3 + xy^2 - 2x^2 - 2y^2 + x = 0$$

and asymptote $x = 2$.

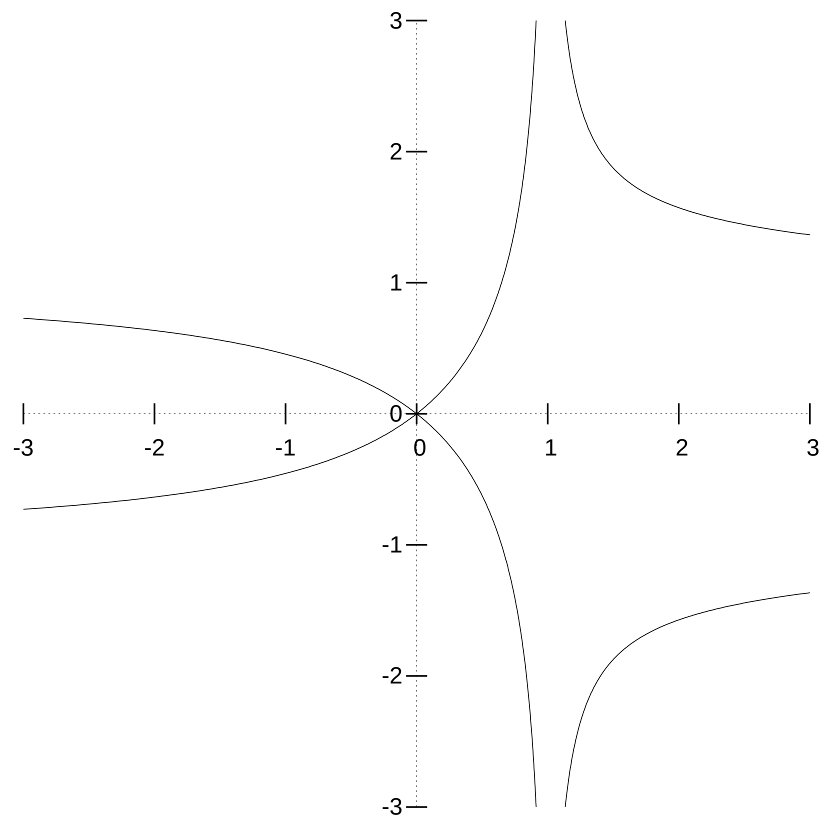


12. $r = \sec(\theta) + \cot(\theta)$

This curve has Cartesian equation

$$(x^2 + y^2)y^2(x-1)^2 - x^4 = 0$$

and asymptotes $y = \pm 1$.

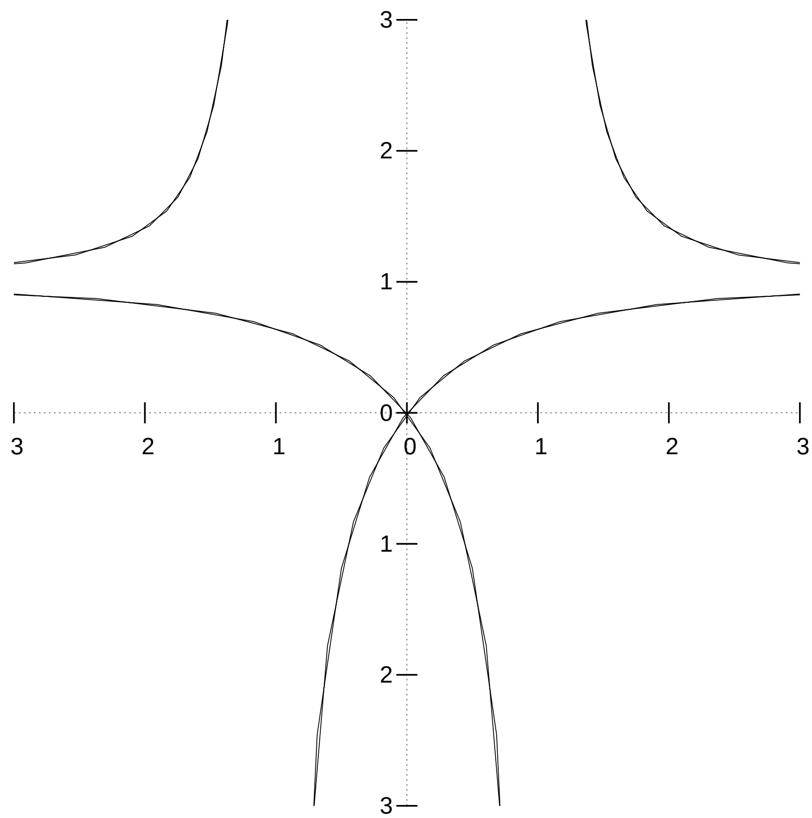


13. $r = \csc(\theta) + \tan(\theta)$

This curve has Cartesian equation

$$y^4 - (x^2 + y^2)x^2(y-1)^2 = 0$$

and asymptotes $x = \pm 1$ and $y = \pm 1$.

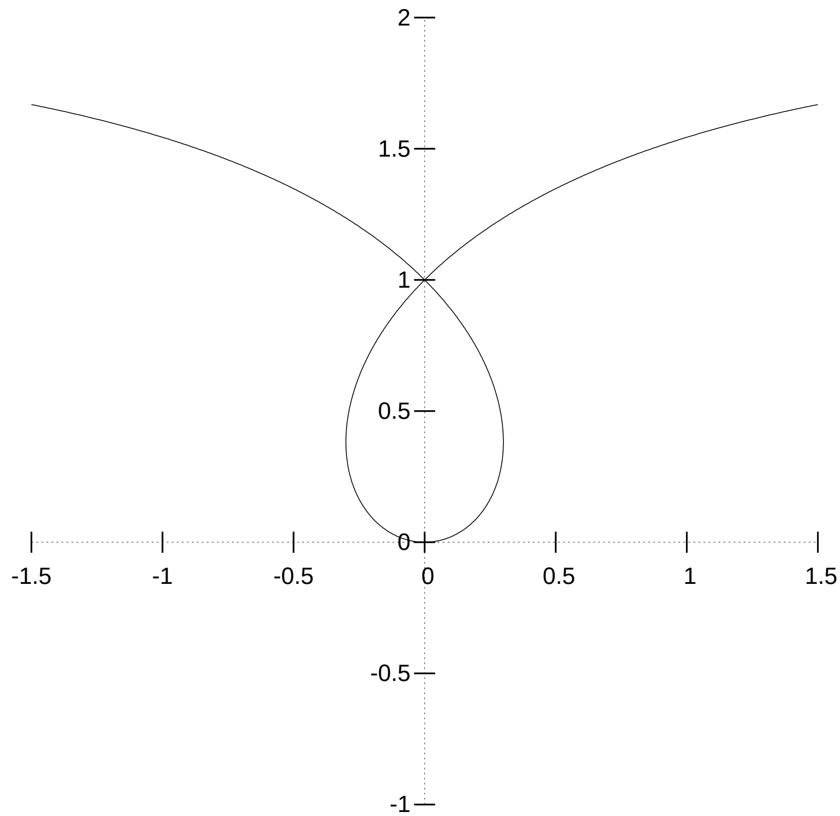


14. $r = \csc(\theta) + \cot(\theta)$

This curve has equation

$$(x^2 + y^2)(y - 1)^2 - x^2 = 0$$

and asymptote $y = 2$.

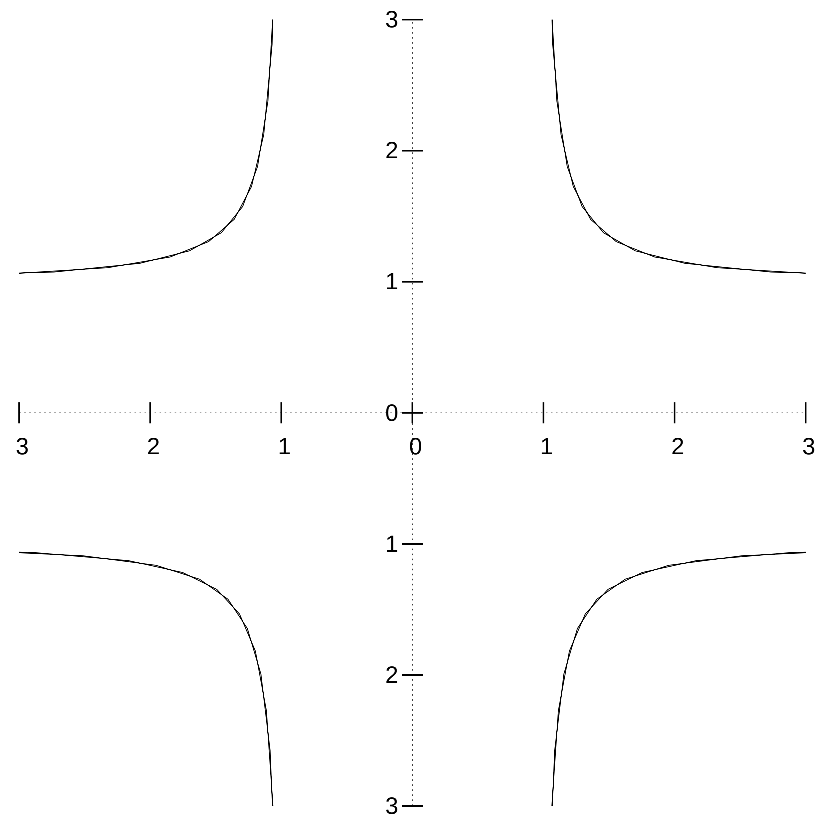


15. $r = \tan(\theta) + \cot(\theta)$

This curve has Cartesian equation

$$x^2(y^2 - 1) - y^2 = 0$$

and asymptotes $x = \pm 1$ and $y = \pm 1$.



Exercises:

1. Verify the cartesian equations given in this section.
2. A number of pairs of curves in this section are simple rotations of each other; explain.

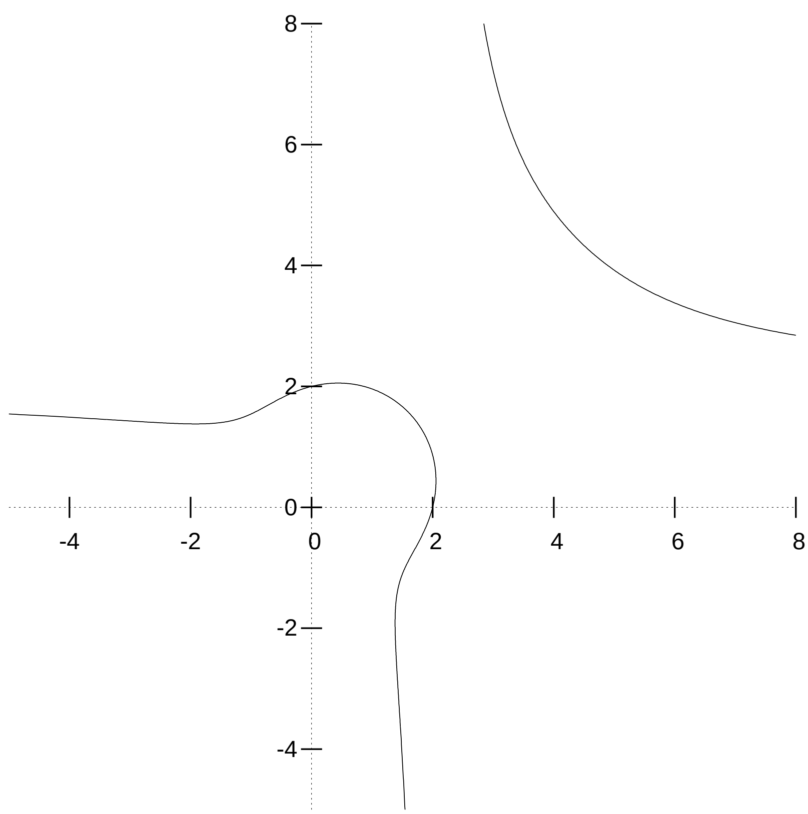
Other examples

1. $r = \cos(\theta) + \sin(\theta) + \sec(\theta) + \csc(\theta) + \cot(\theta) + \tan(\theta)$

   This curve has Cartesian equation

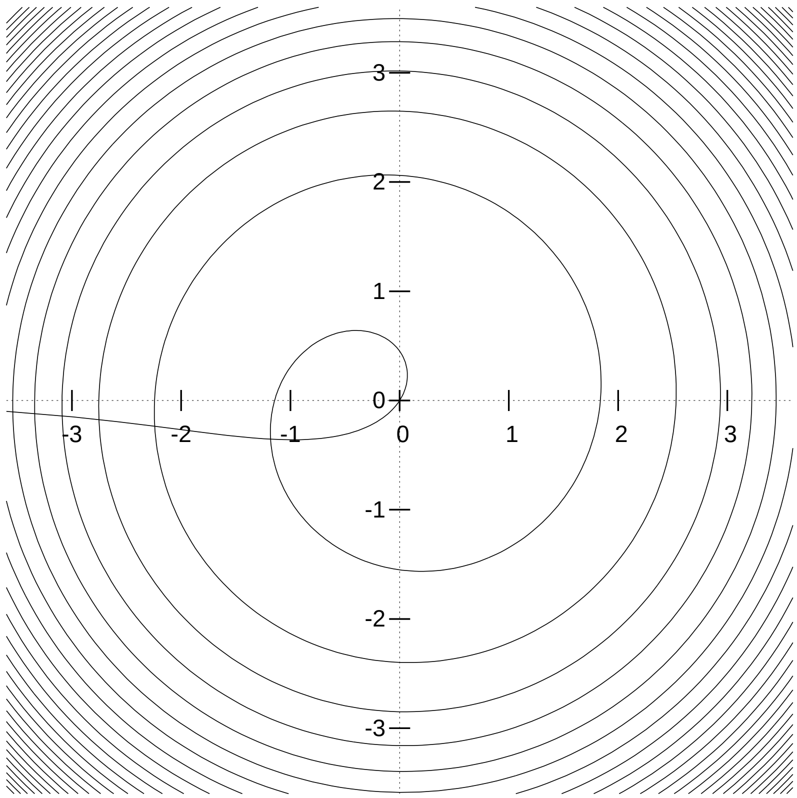
   $$((x^2 + y^2)(xy - x - y) - xy(x + y))^2 - (x^2 + y^2)^3 = 0$$

   and asymptotes $x = 2$ and $y = 2$.



2. $r = \ln\theta$

This is a spiral.



Exercises:

(a) Explain why the spiral gets more and more tightly wound as it goes farther from the origin.

(b) Explain the little "tail" that causes all of the self-intersections in the third quadrant.

(c) Find the exact location of the self-intersection nearest the origin.

3. $r = \theta + \frac{1}{\theta}$