Homework 3 - Math 381 A - Winter 2017 - Dr. Matthew Conroy

1. Consider the following game played with a single, fair, 6-sided die.

   A player starts with a score of zero and throws the die repeatedly, adding the face that appears on each throw of the die to their score.

   If the score is ever a prime number less than 18 $(2, 3, 5, 7, \dots)$, then the player loses.

   If the score reaches a value of 18 or greater, the player wins.

   Model this game using a Markov chain. Set up a transition matrix with two absorbing states (one for winning, one for losing). Use the transition matrix to determine the probability of the player winning the game. Be sure to thoroughly support all of your claims.

   Your calculations should be exact (i.e., the probability should be expressed as an integer fraction, not as a decimal approximation). You may use a system (like Sage, PARI/GP, Maple, etc.) that allows you to perform the calculations exactly. Alternatively, figure out a way to perform the calculations entirely with integer operations which can be done exactly on more systems.

2. Consider the Markov chain with states $0, 1, 2,$ and 3 diagramed at right. The diagram shows all possible one-step transitions, labelled with the probability.

   Transitions with zero probability are omitted from the diagram.

   Determine the limiting distribution of this chain.

   In the long run, what percentage of the time is the chain in each state?