Homework 2 - Math 381 A - Autumn 2017 - Dr. Matthew Conroy

Note: If you write any code to help you solve any of the homework problems in this course, you must include a printed version of the code (thoroughly commented) with your homework submission. The same is true for all software input and output. Please do not include screenshots or photographs of your computer screen. If necessary, copy and paste input and/or output to a text editor and print from there. This note applies to all homework in this course, and is part of the course homework guidelines.

1. Let G be a graph with adjacency matrix

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

   Use the fact that the ij-th entry of A^k gives the number of walks (defined in problem 2, below) of length k from vertex i to vertex j in G to argue whether or not G is connected. Drawing a picture of the graph is not sufficient.

2. Let a and b be positive integers. We say that a divides b iff $b = ak$ for some integer k.

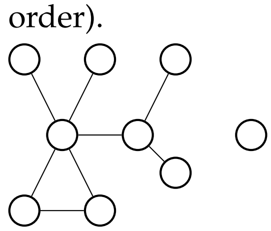
   So 2 divides 10, and 3 does not divide 8.

   Let $V = \{2, 3, 4, \ldots, 22\} = \{j \in \mathbb{Z} : 2 \leq j \leq 22\}$.

   Define a graph H with V as its vertex set and edge set E defined by $(v_1, v_2) \in E$ iff $v_1 \neq v_2$ and v_1 divides v_2 or v_2 divides v_1. So $(2, 6)$ is an edge in H; $(3, 4)$ is not.

   Recall that the edge $(2, 6)$ is the same as the edge $(6, 2)$.

   (a) The *degree sequence* of a graph G is the sequence of the degrees of all vertices in G. For example, the graph below has degree sequence $5, 3, 2, 2, 1, 1, 1, 1, 0$ (in decreasing order).

   Give the degree sequence of H in decreasing order.

   (b) A *path* from vertex u to vertex v in a graph G is an alternating sequence of vertices and edges

$$u = u_0, e_1, u_1, e_2, \ldots, u_{n-1}, e_n, u_n = v$$

   where $e_i = (u_{i-1}, u_i)$ and none of the vertices are repeated.

(A *walk* is the same as a path except that we allow repetition of edges and vertices.)

The *length* of a path is the number of edges in the path.

The *distance* from vertex u to vertex v in a graph G is the length of the shortest path from u to v.

A graph G is connected if, for all pairs of vertices u and v in G, there is a path from u to v.

Remove the vertices that have degree zero from H (along with their incident edges), to get the *subgraph* H'. Is H' connected? Support your claim fully.

Which pair(s) of vertices in H' are farthest apart? Support your claim fully.