

This is an example of how to write up a homework problem in Math 381. Text in red is commentary, and not part of what you should write.

Suppose the homework problem is the following:

1. Let  $G$  be the graph on seven vertices with adjacency matrix  $A$  defined by

$$A_{ij} = 1 \text{ iff } \cos(i + 3j) > 0.$$

Determine whether or not  $G$  is connected.

The key things are to make sure that your answer clearly shows what the problem is, and defines all objects needed for the problem.

Your answer could look like this:

1. We want to determine whether or not the graph  $G$  is connected, where  $G$  has adjacency matrix defined by

$$A_{ij} = 1 \text{ iff } \cos(i + 3j) > 0, 0 \leq i, j \leq 7.$$

Using PARI/GP, we can generate  $A$  like this:

It is helpful to set code apart so that it is distinct from the rest of your writing. Here, I used horizontal rules, and the LaTeX package *listings* to give the code a different look.

---

```
A=matrix(7,7);\nfor(i=1,7,for(j=i+1,7,if(cos(i+3*j)>0,A[i,j]=1;A[j,i]=1)))
```

---

The result is the matrix  $A$ :

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

The  $ij$ -th entry of  $A^k$ , where  $k$  is a positive integer, indicates the number of walks of length  $k$  from vertex  $i$  to vertex  $j$ .

In particular, the  $ij$ -th entry of  $A^k$  will be nonzero if and only if there is a path of length  $k$  from vertex  $i$  to vertex  $j$ .

Looking at various  $A^k$ , we found that, with  $k = 3$ , we got:

We want to use fact that  $A^3$  has no zero entries, so we should *show*  $A^3$  to convince the reader of this fact.

$$A^3 = \begin{pmatrix} 4 & 6 & 4 & 7 & 3 & 5 & 3 \\ 6 & 4 & 8 & 10 & 3 & 6 & 3 \\ 4 & 8 & 2 & 3 & 7 & 1 & 7 \\ 7 & 10 & 3 & 4 & 8 & 2 & 8 \\ 3 & 3 & 7 & 8 & 4 & 3 & 5 \\ 5 & 6 & 1 & 2 & 3 & 2 & 3 \\ 3 & 3 & 7 & 8 & 5 & 3 & 4 \end{pmatrix}$$

Since all entries of  $A^3$  are non-zero, there exists a walk of length 3 between every pair of vertices.

Be to answer precisely the question that was asked.

Since there is a walk between every pair of vertices,  $G$  is connected.