Homework 5 - Math 381 A - Winter 2016 - Dr. Matthew Conroy

1. Consider a variation of Buffon's needle experiment. For this version, suppose the plane is crossed with parallel lines in both horizontal and vertical directions, and each resulting square is crossed with one diagonal.

   Write a simulation to experimentally estimate the probability that a randomly thrown needle will cross a line (either a diagonal, or one of the set of parallel lines). Describe your process in developing your code (separately from code comments, and in complete sentences).

   Experiment with two needle lengths: one equal to the width between adjacent pairs of the main parallel lines, and another equal to one-quarter that length.

   Include code output and plot the estimate of the probability versus the number of iterations for several runs of each simulation (run it long enough that you can see some asymptotic behavior).

   Discuss the estimates you get: what is a range of values that you are confident that the actual probability lies in? Why? How confident are you?

   Include the code in your writeup, what machine you ran it on, and how long it took to run the simulations.

2. The following problems concern Poisson processes.

   (a) Consider a Poisson process with rate $\lambda = 1$ (so the event occurs, on average, one time per unit of time (e.g., once per hour)). Find the length of a time interval such that the probability of 0,1 or 2 events occurring in that interval is 0.99. Be sure your answer is accurate to at least 8 digits of precision.

   (b) Consider a Poisson process with rate $\lambda = 1$ (so the event occurs, on average, one time per unit of time (e.g., once per hour)). Suppose $t = m$ is an integer multiple of the time unit. Show that the probability of m occurrences is equal to the probability of $m - 1$ occurrences in the time interval.

   (c) Consider a Poisson process with rate λ. How small does a time interval need to be such that the probability of two events occurring is less than the probability of one event occurring (your result will be a function of λ)?

   (d) Consider a Poisson process with rate λ. For the purposes of simulation, it can be useful to use interval lengths on which the probability of two events occurring is small compared to the probability of one event occurring. Find the length of the time interval on which the probability of two events occurring is one one-hundredth the probability of one event occurring (your result will be a function of λ).

   (These last two questions illustrate why we often say we like λt to be small for purposes of simulations and approximation.)