

Theorem: Let a and b be positive integers. Suppose $a|b$. Then $b \geq a$.

Proof: Let a and b be positive integers and suppose $a|b$.

Then $\exists k \in \mathbb{Z}$ such that $b = ak$.

Suppose $k = 0$.

Then $b = a \cdot 0 = 0$ by EPI #1.

But $b > 0$, so this is a contradiction.

Hence, the assumption that $k = 0$ is false.

Thus, $k \neq 0$.

Suppose $k < 0$.

Then $ak < a \cdot 0$ by EPI #10.

That is, $ak < 0$ by EPI #1.

So $b < 0$.

This is a contradiction, since $b > 0$.

Hence, the assumption that $k < 0$ is false.

Thus, $k \not< 0$.

Hence, $k > 0$.

Suppose $k = 1$.

Then $b = a$.

Suppose $k \neq 1$.

Then $k > 1$.

So $ak > a$ by EPI #10.

That is, $b > a$.

Thus, $b \geq a$. ■

Note the contrapositive: if $b < a$, then $a \nmid b$.

That is, if $b < a$ then a does not divide b .

So, feel free now to make statements like "Since $1 < 4$, $4 \nmid 1$."