

Homework 7 - Math 300 C - Spring 2016 - Dr. Matthew Conroy

Relevant reading: Velleman 7.1, 7.2

1. For each of the following pairs of sets, give a bijection from the first set to the second set. Prove that your bijection *is* a bijection. Then give the inverse of each bijection.
 - (a) The set of positive integers that are congruent to 3 modulo 7 and the set of positive integers that are congruent to 11 modulo 19
 - (b) $(-2, \infty)$ and $(-\infty, 7)$ (these are intervals of real numbers)
 - (c) $(-\infty, 3)$ and $(0, 1)$ (these are intervals of real numbers)
2. Prove the following theorems.
 - (a) Let A and B be finite sets. If $A \cap B = \emptyset$, then $|A \cup B| = |A| + |B|$.
 - (b) For any finite sets A and B , $|A \setminus B| + |A \cap B| = |A|$.
 - (c) For any finite sets A and B , $|A \cup B| = |A| + |B| - |A \cap B|$.
3. Let A be a finite set. Prove that if $f : A \rightarrow A$ is injective, then f is bijective.
4. Prove that, if $A \sim B$, then $\mathcal{P}(A) \sim \mathcal{P}(B)$.
5. Let n be a positive integer. Use induction to prove that the union of n pair-wise disjoint countable sets is countable.